

**B.SC. DEGREE END SEMESTER EXAMINATION OCTOBER 2016**  
**SEMESTER - 5: MATHEMATICS (CORE COURSE)**  
**COURSE: U5CRMAT8 - FUZZY MATHEMATICS**

Time: Three Hours

Max. Marks: 75

**SECTION A***Answer all questions*

1. Define a fuzzy set.
2. Define the scalar cardinality of a fuzzy set
3. Define cut worthy property
4. Give an example of a fuzzy complement
5. State second characterization theorem of fuzzy complements.
6. Give the pseudo-inverse of an increasing generator  $g$
7. Define the Archimedean t-norm
8. Define the product  $[a, b].[c, d]$
9. Write the canonical form of an unconditional and qualified proposition
10. Define the primitive  $a \Rightarrow b$  according to Lukasiewicz

(1 x 10 = 10)

**SECTION B***Answer any eight*

11. For  $A \in F(X)$  and  $\alpha \in [0,1]$ , Show that  ${}^\alpha(A) = (1-\alpha) + (A)$
12. Prove that a continuous fuzzy complement has a unique equilibrium
13. Let  $A, B \in F(X)$  and  $f: X \rightarrow Y$  be an arbitrary crisp function. Prove by extension principle that  $A \subseteq B$  then  $f(A) \subseteq f(B)$
14. Prove that  $i(a, b) \leq \min(a, b)$  for any t-norm  $i$
15. Prove that any real number and any closed interval are fuzzy numbers
16. State the characterization theorem of t-norms
17. Show that standard fuzzy union is the only idempotent t-conorm.
18. Define linguistic Hedges with an example
19. Show that  $X=B-A$  is not the solution of the fuzzy equation  $A+X=B$ .
20. Define the fuzzy numbers  $\text{MIN}(A,B)$  and  $\text{MAX}(A,B)$  where  $A$  and  $B$  are fuzzy numbers

(2 x 8 = 16)

**SECTION C***Answer any five*

21. Let  $A_i \in F(X)$  for all  $i \in I$ , where  $I$  is an index set. Prove that  $\bigcap_{i \in I} A_i$  and that  $\bigcup_{i \in I} A_i$ .
22. State and prove the first decomposition theorem of fuzzy sets.
23. Show that the triples  $\langle i_{\min}, u_{\max}, c \rangle$  are dual with respect to any fuzzy complement  $c$
24. Prove that the algebraic product  $i(a, b) = ab$ , where  $a, b \in [0, 1]$  is a t-norm

25. Show that a fuzzy set A on R is convex if and only if  $A[\lambda x_1 + (1-\lambda)x_2] \geq \min[A(x_1), A(x_2)]$  for all  $x_1, x_2 \in R$  and all  $\lambda \in [0,1]$ .

26. Given an involute fuzzy complement c and an increasing generator g of c, prove that t-norm and

t-conorm generated by g are dual with respect to c.

27. Explain unconditional and unqualified propositions.

( 5 x 5 = 25)

### SECTION D

Answer **any two**

28. (i) Prove that  $c : [0,1] \rightarrow [0,1]$  defined by  $c(a) = \begin{cases} 1, & \text{for } a \leq t \\ 0, & \text{for } a > t \end{cases}$ , is not continuous and involutive

(ii) Define the Sugeno Class of fuzzy complements and prove that it is involutive

(iii) Define a decreasing generator f and its pseudo inverse  $f^{(-1)}$ . Also define their compositions

$$f \circ f^{(-1)} \text{ and } f^{(-1)} \circ f$$

29. State and prove the necessary and sufficient condition for a fuzzy set A defined on R to be a fuzzy number.

30. What are fuzzy quantifiers? Explain the two kinds of fuzzy quantifiers with examples

31. Explain the three inference rules with examples

(12 x 2 = 24)

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