

Reg. No..... Name.....

B.Sc. DEGREE END SEMESTER EXAMINATION OCTOBER 2016
SEMESTER - 5 : CORE COURSE FOR B. SC MATHEMATICS
COURSE: U5CRMAT7 - ABSTRACT ALGEBRA

Time: Three Hours

Max Marks: 75

PART - A(Answer **all** questions. Each question carries 1 mark.)

1. On Q^+ define $a * b = \frac{ab}{2}$. Find the inverse of a .
2. What is the number of binary operations on a set having 3 elements?
3. What is the order of $\sigma = (4, 5)(2, 3, 7)$?
4. Find the number of generators of the cyclic group Z_8 .
5. Give an example of a finite group which is not cyclic.
6. What is the order of $\frac{Z_6}{3\mathbb{Z}}$?
7. Find all units in the ring Z .
8. Find all divisors of zero in the ring Z_{12} .
9. What is the characteristic of the ring $Z \times Z$.
10. Q is an ideal in R . Say True or False.

(1 x 10 = 10)

PART - B(Answer **any eight**. Each question carries 2 marks.)

11. Let G be a group such that $a^2 = e$ for all a in G . Show that ZG is abelian.
12. Write all subgroups of the group Z_4 .
13. Find the product of cycles $(1, 3, 2, 7)$ and $(4, 8, 6)$ that are permutations of $(1, 2, 3, 4, 5, 6, 7, 8)$.
14. Find the number of elements in the cyclic subgroup of Z_{4^2} generated by 30.
15. Show that every cyclic group is abelian.
16. Prove that factor group of an abelian group is abelian.
17. If R is a ring with additive identity O then for $a \in R$. Prove that $0.a = a.0 = 0$.
18. Solve the equation $x^2 - 5x + 6 = 0$ in Z_{12} .
19. Find the characteristic of $Z_6 \times Z_{15}$.
20. If R is a ring with unity and N is an ideal of R containing a unit. Show that $N = R$.

(2 x 8 = 16)

PART - CAnswer (**any five**. Each question carries 5 marks.)

21. Prove that the identity element and inverse of each element are unique in a group.
22. Prove that every permutation of a finite set is a product of disjoint cycles.

23. Prove that subgroup of a cyclic group is cyclic.
24. Prove that an infinite cyclic group is isomorphic to the group Z of integers under addition.
25. In z_n prove that the divisors of 0 are precisely, those elements which are not relatively prime to n .
26. Prove that every finite integral domain is a field.
27. If I_1 and I_2 are two ideals of a ring R . Then prove that $I_1 \cap I_2$ is an ideal of R .
(5 x 5 = 25)

PART - D

(Answer **any two**. Each question carries 12 marks.)

28. a) Let A be a non-empty set. Let S_A be the collection of all permutations on A . Then prove that S_A is a group under permutation multiplication.
b) For $n \geq 2$. Prove that the number of even permutations in S_n is same as the number of odd permutations.
29. State and prove Cayley's theorem.
30. State and prove fundamental homomorphism theorem.
- 31 a) If R is a ring with unity 1. Then prove that R has characteristic $n > 0$ if and only if n is the least positive integer such that $n \cdot 1 = 0$.
b) Define left ideal of a ring. Find a sub ring of the ring $z \times z$. that is not an ideal of $z \times z$.

(12 x 2 = 24)
