Reg. No.................................... Name.................................. **U524**

B.Sc. DEGREE END SEMESTER EXAMINATION OCTOBER 2016

SEMESTER – 5: CORE COURSE FOR B. SC MATHEMATICS

COURSE: U5CRMAT7 – ABSTRACT ALGEBRA

Time: Three Hours Max Marks: 75

PART – A

Answer all questions. Each question carries 1 mark.

1. On Q^+ define a*b = $\frac{ab}{2}$. Find the inverse of a.
2. What is the number of binary operations on a set having 3 elements?
3. What is the order of σ = (4, 5) (2, 3, 7)?
4. Find the number of generators of the cyclic group Z_8.
5. Give an example of a finite group which is not cyclic.
6. What is the order of $\frac{Z_6}{¿3>¿¿}$
7. Find all units in the ring Z.
8. Find all divisors of zero in the ring Z_{12}.
9. What is the characteristic of the ring Z x Z.
10. Q is an ideal in R. Say True or False. (1 x 10 = 10)

PART – B

Answer any **Eight**. Each question carries 2 marks.

11. Let G be a group such that $a^2 = e$ for all a in G. Show that ZG is abelian.
12. Write all subgroups of the group Z_4.
13. Find the product of cycles (1, 3, 2, 7) and (4, 8, 6) that are permutations of (1, 2, 3, 4, 5, 6, 7, 8).
14. Find the number of elements in the cyclic subgroup of z_{4^2} generated by 30.
15. Show that every cyclic group is abelian.
16. Prove that factor group of an abelian group is abelian.
17. If R is a ring with additive identity O then for $a \in R$. Prove that $0.a = a.0 = 0$.
18. Solve the equation $x^2 - 5x + 6 = 0$ in z_{12}.
19. Find the characteristic of $z_6 \times z_{15}$
20. If R is a ring with unity and N is an ideal of R containing a unit. Show that $N = R$.

(2 x 8 = 16)

PART – C

Answer any **Five**. Each question carries 5 marks.

21. Prove that the identity element and inverse of each element are unique in a group.
22. Prove that every permutation of a finite set is a product of disjoint cycles.
23. Prove that subgroup of a cyclic group is cyclic.

24. Prove that an infinite cyclic group is isomorphic to the group Z of integers under addition.

25. In z_n prove that the divisors of 0 are precisely, those elements which are not relatively prime to n.

26. Prove that every finite integral domain is a field.

27. If I_1 and I_2 are two ideals of a ring R. Then prove that $I_, \cap I_2$ is an ideal of R.

(5 x 5 = 25)

PART – D

Answer any **Two**. Each question carries 12 marks.

28. a) Let A be a non-empty set. Let S_A be the collection of all permutations on A. Then prove that S_A is a group under permutation multiplication.

    b) For $n \geq 2$. Prove that the number of even permutations in S_n is same as the number of odd permutations.

29. State and prove Cayley's theorem.

30. State and prove fundamental homomorphism theorem.

31 a) If R is a ring with unity 1. Then prove that R has characteristic $n > 0$ if and only if n is the least positive integer such that n.1 = 0.

    b) Define left ideal of a ring. Find a sub ring of the ring $z \times z$ that is not an ideal of $z \times z$.

(12 x 2 = 24)

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