

B.SC DEGREE END SEMESTER EXAMINATION OCTOBER 2016
SEMESTER - 5: MATHEMATICS (CORE COURSE)

COURSE: U5CRMAT5, U5CRCMT5 - MATHEMATICAL ANALYSIS

(Common for B Sc Mathematics & B Sc Computer Application)

Time: Three Hours

Max. Marks: 75

PART-A(Answer **all** questions. Each question carries 1 mark)

1. Find the Supremum of $\left\{1 + \frac{(-1)^n}{n}; n \in \mathbb{N}\right\}$
2. State order completeness in \mathbb{R}
3. Give an example of an open set which is not an interval
4. Obtain the derived set of $\{x: 1 < x < 2\}$
5. Define a perfect set & give an example
6. Give an example of a bounded sequence
7. Write the limit points of the sequence $\{1 + (-1)^n\}; n \in \mathbb{N}$
8. Give an example of a monotonic increasing sequence which is bounded above
9. Show that for any complex number $z; \operatorname{Re}(iz) = -\operatorname{Im} z$
10. Write $\frac{4+i}{2-3i}$ in the form $a+ib$

(1 x 10 = 10)

PART-B(Answer **any eight** questions. Each question carries 2 marks)

11. Prove that the greatest number of a set if it exists is the supremum of the set
12. State Dedekind's property of real numbers
13. If M & N are nbds of a point x then show that $M \cap N$ is also a nbd of x
14. If S & T are subsets of \mathbb{R} show that $S \subseteq T \Rightarrow S' \subseteq T'$
15. S is a set such that $T = \mathbb{R} - S$ is open, show that S is closed
16. Prove that the set of all odd positive integers is countably infinite
17. Show that every convergence sequence is bounded
18. Show that $\{S_n\}$ where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ cannot converge
19. Sketch the points determined by $\operatorname{Re}(z-i) = 2$
20. Find the principal argument of $z = \frac{-2}{1+i\sqrt{3}}$

(2 x 8 = 16)

PART-C(Answer **any five** questions. Each question carries 5 marks)

21. Prove that the order completeness property of real numbers implies Dedekind's property
22. Prove that the interior of a set S is the largest open sub set of S
23. Prove that the set of all rational numbers in $[0, 1]$ is countable
24. Prove that every bounded sequence with a unique limit point is convergent
25. State & prove Sandwich theorem

26. Show that the sequence $\{S_n\}$ where $S_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}$ is convergent

27. If c is any n^{th} root of unity other than unity itself, then show that $1+c+c^2+c^3+\dots+c^{n-1} = 0$

(5 x 5 = 25)

PART-D

(Answer **any two** questions. Each question carries 12marks)

28. Prove that the set of rational numbers is not order complete
29. State & prove Bolzano Weierstrass theorem (for sets)
30. Show that the sequence $\{r^n\}$ converges iff $-1 < r \leq 1$

31. (a) Show that $\lim_{n \rightarrow \infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}} \right] = 1$

(b) Show that the sequence $\{a_n\}$ where $a_n = \frac{(3n)!}{(n!)^3}$ converges

(12 x 2 = 24)
