Reg.No.. Name:.................................... **U430**

BSc DEGREE END SEMESTER EXAMINATION MARCH 2016
SEMESTER – 4, COMPLEMENTARY COURSE FOR BSC PHYSICS AND CHEMISTRY
COURSE: U4CPMAT04, FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND ABSTRACT ALGEBRA

Time: Three Hours   Max. Marks: 75

PART A
(Answer **all** questions. Each question carries 1 mark)

1. What is the fundamental period of $\sin 2x$?
2. Is $e^{-|x|}, -\pi < x < \pi$ an odd or even function. Why?
3. Write Legendre's differential equation.
4. What are the direction cosines normal to the surface $z = f(x, y)$ at the point (x, y, z).
5. What is the order of the differential equation $\left(\dfrac{\partial \theta}{\partial x}\right)^3 + \dfrac{\partial \theta}{\partial t} = 0$.
6. Give an example of a first order nonlinear partial differential equation.
7. Find the relative error of π if the approximate value is 3.142857 and its true value 3.1415926.
8. Give an example of an algebraic function.
9. Define a semi group.
10. What is the order of the symmetric group S$_4$.

(1 x 10 = 10)

PART B
(Answer any **eight** questions. Each question carries 2 marks)

11. Sketch the graph of the function $f(x) = \begin{cases} -1, -\pi < x < 0 \\ 1, 0 < x < \pi \end{cases}$, $f(x + 2\pi) = f(x)$.
12. Write the expressions for P$_0$(x), P$_2$(x) and P$_4$(x).
13. Show that $\lceil v+1 = v \lceil v \rceil \rceil$ where v is a nonnegative real number.
14. Eliminate the arbitrary constants a and b from $z = ax + by + ab$ and form a partial differential equation.
15. Form the differential equation by eliminating the arbitrary function f from $z = f(x^2 - y^2)$.
16. If $u = 3v^7 - 6v$, find the percentage error in u at $v = 1$, if the error in v is 0.05.
17. Write down the computational steps in the bisection method for finding the root of the equation $f(x) = 0$.
18. Use the method of iteration to find a real root of $2x - \log_{10} x = 7$
19. Let G be a group, a ε G, Then show that $H = \{a^n : n \in Z\}$ is a subgroup of G.
20. Describe all ring homomorphism from Z into Z.

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PART C
(Answer any **five** questions. Each question carries 5 marks)

21. A sinusoidal voltage $E \sin\omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting function

$$u(t) = \begin{cases} 0 \; if -L < x < 0 \\ E\sin\omega t \; if \; 0 < t < L \end{cases} \quad p = 2L = \frac{2\pi}{\omega}, \; L = \frac{\pi}{\omega}.$$

22. Find series solution of the equation $(1 - x¿¿2)y'' - 2xy' + 2y = 0¿.$

23. Find the integral curves of $\dfrac{dx}{cy - bz} = \dfrac{dy}{az - cx} = \dfrac{dz}{bx - ay}.$

24. Solve $(y + zx)p - (x + yz)q = x^2 - y^2$

25. Use the method of false position to obtain a root of $x^3 - 4x + 1 = 0.$

26. Obtain a root correct to 5 decimal places for $2x^3 - 3x - 6 = 0$ using the Newton Raphson method.

27. Check whether the set { (1, 0, 1), (2, 1, 4), (0, 0, 1)} is a basis for R^3 over R?

(5 x 5 = 25)

Part D
(Answer any **two** questions. Each question carries 12 marks)

28. a) Find two half range expansions of the functions $f(x) = x, 0 < x < 2.$

    b) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}} \sin x.$

29. a) Find the integral curves of $\dfrac{dx}{x^2} = \dfrac{dy}{y^2} = \dfrac{dz}{(x + y)z}.$

    b) Find the general solution of $px(z - 2y^2) = (z - qy)(z^2 - y^2 - 2x^3)$

30. a) Find the real roots of the equation $2x = \cos x + 3$ correct to 3 decimal places using iterative method.

    b) Find the real roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ using the quotient-difference method.

31. a) Show that a finite dimensional vector space has a basis.

    b) State and prove Cayley's theorem

(12 x 2 = 24)

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