

Reg.No..... Name:.....

U415

BSc DEGREE END SEMESTER EXAMINATION MARCH 2016

SEMESTER - 4:

CORE COURSE FOR MATHS AND COMPUTER APPLICATIONS

COURSE: U4CRMAT04-U4CRCMT04 - VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

Time: Three Hours

Max. Marks: 75

Part A

Answer **all** questions. Each question carries 1 mark.

1. Find the equation of a plane through $(0, 2, -1)$ and normal to $\mathbf{n} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$.
2. Find the angle between the planes $x + y = 1$ and $2x + y - 2z = 2$.
3. Find the length of one turn of the helix $r(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t \mathbf{k}$.
4. Find the gradient of $f(x, y) = \frac{x^2}{4} + y^2$ at $(-2, 1)$.
5. Check whether $ydx + xdy + 4dz$ is exact
6. Is $\mathbf{F} = 12xz^2 \mathbf{i} - 9yz^2 \mathbf{j} - z^3 \mathbf{k}$ solenoidal?
7. State the fundamental theorem of algebra.
8. Find the minimum number of possible imaginary roots of $12x^7 - x^4 + 10x^3 - 28 = 0$.
9. Give an example of a transcendental equation.
10. Write Newton-Raphson formula.

(1 x 10 = 10)

Part B

Answer any **eight** questions. Each question carries 2 marks.

11. Find the equation of a plane through $(1, 1, -1), (2, 0, 2), (0, -2, 1)$.
12. Find the unit tangent vector to the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$ at a point t .
13. Find the derivative of $f(x, y) = x^2 + xy$ at $(1, 2)$ in the direction of $\mathbf{i} + \mathbf{j}$.
14. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to the point $(1, 1, 1)$.
15. Find the circulation of the field $\mathbf{F} = (x - y) \mathbf{i} + x \mathbf{j}$ around the circle $r(t) = (\cos t) \mathbf{i} + t \mathbf{j}, 0 \leq t \leq 2\pi$.
16. Check whether $\mathbf{F} = (2x - 3) \mathbf{i} - z \mathbf{j} + \cos z \mathbf{k}$ is conservative?

17. Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in arithmetic progression

18. If α, β, γ are the roots of $x^3 + 5x^2 + 3x + 2 = 0$. Find $\sum \frac{1}{\alpha}$.

19. Find a real root $x^3 + x - 1 = 0$ between 0 and 1 by Regula-falsi method.

20. Find an approximate root of the equation $x^3 - 9x + 1 = 0$ between 2 and 3 by bisection method in four steps.

(2 x 8 = 16)

Part C

Answer any **five** questions. Each question carries 5 marks.

21. Find the normal and tangential scalar components of acceleration of the motion

$$r(t) = (\cos t + t \sin t) \mathbf{i} + (\sin t - t \cos t) \mathbf{j}, t > 0.$$

22. Find the tangent plane and the normal line of the surface

$$f(x, y, z) = x^2 + y^2 + z - 9 = 0 \text{ at}$$

the point $P_0 (1, 2, 4)$.

23. Use Green's Theorem to evaluate the integral, $\oint_C xy dy - y^2 dx$ where C is the square cut

from the first quadrant by the lines $x = 1$ and $y = 1$.

24. Evaluate the integral $\iint_S (7xi - zk) \cdot nd\sigma$ over the sphere $S: x^2 + y^2 + z^2 = 4$ by divergence theorem.

25. Solve $24x^3 - 14x^2 - 63x + 45 = 0$ given that one root is twice another root.

26. Find the equation whose roots are 2 less than the roots of the equation

$$x^4 - 5x^3 + 7x^2 - 4x + 5 = 0.$$

27. Find the root of the equation $x^3 + x - 1 = 0$ by iteration method near $x = 1$

(5 x 5 = 25)

Part D

Answer any **two** questions. Each question carries 12 marks.

28. Verify Stoke's theorem for the hemisphere $x^2 + y^2 + z^2 = 9, z \geq 0$, whose bounding circle is

$$x^2 + y^2 = 9, z = 0 \text{ and the field } \mathbf{F} = y \mathbf{i} - x \mathbf{j}.$$

29. Find the area of the surface cap cut from the hemisphere

$$x^2+y^2+z^2=2, z \geq 0 \text{ by the}$$

$$\text{cylinder } x^2+y^2=1 .$$

30. Solve the equation $x^3-9x+28=0$ by Cardan's method.

31. Use Newton-Raphson method to obtain a root correct to 3 decimal places for the equation

$$x \sin x + \cos x = 0 \text{ near } x = 3$$

(12 x 2 = 24)
