# M.SC DEGREE END SEMESTER EXAMINATION OCTOBER 2016 SEMESTER - 3: MATHEMATICS <br> COURSE: P3MATT15- OPTIMIZATION TECHNIQUES 

Common for Regular (2015 Admission) \& Supplementary / Improvement (2014 Admission)

Time: Three Hours

## SECTION A

Answer any five questions. Each question carries 2 marks

1. Differentiate between an ILPP and an MILPP
2. Formulate the following knapsack problem as an ILP
"There are $n$ objects $j=1,2, \ldots \ldots . . n$, whose weights are $w_{j}$ and values are $v_{j}$.
They have to be chosen and to be packed in a knapsack so that the total value of the objects chosen is maximum subject to the total weight not exceeding W".
3. In sensitivity analysis briefly explain the response to the addition of new variables.
4. Introduce the problem of maximum potential difference.
5. Explain
i) a two person zero sum game
ii) a pay off matrix iii) saddle point of a matrix game
6. Distinguish between a game of chance and a game of strategy with examples.
7. Define convex and concave functions. Explain with the help of a graph.
8. What do you understand by a complementary problem?

$$
(2 \times 5=10)
$$

## SECTION B

Answer any five questions. Each question carries 5 marks
9. Explain the branch and bound algorithm to solve an ILPP
10. What is $0-1$ variable problem and mention the different cases? Formulate the following problem into a 0-1 variable problem
"Maximise $2 x_{1}+5 x_{2}$
Subject to

$$
0 \leq x_{1} \leq 8,0 \leq x_{2} \leq 8 \text { and either } 4-x_{1} \geq 0 \text { or } 4-x_{2} \geq 0 "
$$

11. Find the maximum potential difference $\mathrm{x}_{14}$ between $\mathrm{v}_{1}$ and $\mathrm{v}_{4}$ with the following data
subject to the condition that $\mathrm{X}_{\mathrm{jk}} \leq \mathrm{c}_{\mathrm{jk}}$

| $v$ | 1 |  |  | 3 | 4 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $u$ | $(1,2)$ | $(1,3)$ |  | $(2,3)^{4}$ | $(2,4)$ | $(3,4)$ |

12. Prove that the maximum flow in a network is equal to the minimum of the capacities of all possible cuts in it.
13. The pay off matrix of a game is given below. Write the corresponding linear programming
problem.

$$
\left[\begin{array}{ccc}
1 & -1 & 3 \\
3 & 5 & -3 \\
6 & 2 & -2
\end{array}\right]
$$

14. Solve the following game graphically. The pay off matrix for player $A$ is

B

|  |  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 19 | 15 | 17 | 16 |
|  | 2 | 0 | 20 | 15 | 5 |

15. Minimise $f(x)=3 x^{4}+(x-1)^{2}, \quad 0 \leq x \leq 4$ using golden section search with a resolution of $\varepsilon=0.10$
16. Explain the complementary pivot algorithm to solve a complementary problem.
$(5 \times 5=25)$

## SECTION C

Answer either A or B of each question. Each question carries 10 marks
17. A) Explain cutting plane algorithm to solve an ILPP.
B) Solve by branch and bound method

Minimise $4 x_{1}+5 x_{2}$
Subject to $3 x_{1}+x_{2} \geq 2$

$$
\begin{aligned}
& x_{1}+4 x_{2} \geq 5 \\
& 3 x_{1}+2 x_{2} \geq 7 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

18. A) Tasks $\mathrm{A}, \mathrm{B}, \mathrm{C}, \ldots \ldots . . \mathrm{H}, \mathrm{I}$ constitute a project. The notation $\mathrm{X}<\mathrm{Y}$ means that the task $X$ is to be
finished to begin $Y$. With this $A<D, A<E, B<F, D<F, C<G, C<H, F<I, G<I$
The time in days of completion of each task is as follows.
$\begin{array}{llllllllll}\text { Capacity } & 8 & 10 & 7 & 9 & 16 & 7 & 8 & 14 & 9\end{array}$

Draw a graph to represent the sequence of tasks and find the minimum time of completion
of the project.
B) What is maximum flow in a network? Give an algorithm to find maximum flow
19. A)State and prove the necessary and sufficient condition for the existence of a saddle
point ( $X_{0}, Y_{0}$ ) of $F(X, Y)$
B) State and prove the fundamental theorem of rectangular games
20. A) Using the Kuhn -Tucker conditions solve Minimise $f(x)=x_{1}{ }^{2}-x_{2}$

Subject to

$$
\begin{gathered}
x_{1}+x_{2}=6 \\
x_{1} \geq 1 \\
x_{1}{ }^{2}+x_{2}^{2} \leq 26
\end{gathered}
$$

B) Describe the Hooke and Jeeves search algorithm
$(10 \times 4=40)$

