Reg. No:................................. Name:..

M.SC DEGREE END SEMESTER EXAMINATION OCTOBER 2016

SEMESTER – 3: MATHEMATICS

COURSE: P3MATT13– DIFFERENTIAL GEOMETRY

Common for Regular (2015 Admission) & Supplementary / Improvement (2014 Admission)

Time: Three Hours Max. Marks: 75

Part A

(Answer **any five** questions. Each carriers 2 marks)

1. Show that the graph of any function $f: R^n \to R$ is the level set of some function $F: R^{n+1} \to R$
2. Define an n- surface in R^{n+1}
3. Find the speed of the parameterized curve $\alpha(t) = (\cos 3t, \sin 3t)$
4. Define the Gauss map
5. Compute ∇_v **X** where **X =** $(x_1, x_2, -x_2, x_1)$ and **v** = $(\cos\theta, \sin\theta, -\sin\theta, \cos\theta)$
6. Define Weingarten map
7. Explain the term Normal section of an n- surface S
8. State Inverse function theorem for an n- surface

(2 x 5 = 10)

Part B

(Answer **any five** questions. Each carriers 5 marks)

9. Find the integral curve through P (a ,b) for the vector field **X**(p)= (P, X(P)) where $X(p) = X(x_1,x_2) = (-x_2, x_1)$
10. Show that the two orientations of the n- sphere $x_1^2 + x_2^2 + \dots + x_{n+1}^2 = r^2$ of radius $r > 0$ are given by $\mathbf{N_1}(P) = (P, \frac{p}{r})$ A and $\mathbf{N_2}(p) = (p, -\frac{p}{r})$
11. Prove that $(\mathbf{X} + \mathbf{Y})' = \mathbf{X}' + \mathbf{Y}'$
12. For each a, b, c, d $\in$ R prove that the parameterized curve defined by $\alpha(t) = (\cos(at+b), \sin(at+b), ct+d)$ is a Geodesic on the cylinder $x_1^2 + x_2^2 = 1$ in R^3
13. Find the parameterization of the plane curve and the curvature K(p)of $ax_1 + bx_2 = c$ where $(a, b) \neq 0$ oriented by $\frac{\nabla f}{\|\nabla f\|}$
14. Let $f: U \to R$; $U \sqsubseteq R^{n+1}$ be a smooth function. Define the differential of f , df and prove that it is a differential one-form
15. Find the normal curvature K(**v**) to the surface $x_1 + x_2 + \dots + x_{n+1} = 1$,oriented by

$$\mathbf{N} = \frac{\nabla f}{\|\nabla f\|}$$

16. Let S be a compact connected oriented surface in R^{n+1}.Prove that the Gauss Kronecker curvature K(p) is non zero for all p in S iff the second fundamental form $\psi_p$of S at p is definite for all p ε S

(5 x 5= 25)

Part C

(Answer either part **(a)** or part **(b**). Each question carriers 10 marks)

17. (a) Let **X** be a smooth vector field on an open set U ⊏ R^{n+1} and let p ε U . Prove that there exists an open interval I containing 0 and an integral curve α : I → U of **X** such that

(i) α(0)=p

(ii) If β : Ī →U is any other integral curve of **X** with $(0) = p, then$ Ī ⊂ I and $\beta(t) = \alpha(t) r\ all\ t \in \bar{I}$

(b) Let S be an n surface in R^{n+1},S= f^{-1} (c) where f :U→ R is such that $\nabla f(q) \neq 0$ for all q ε S . Suppose g: U→ R is a smooth function and p ε S is an extreme point of g on S. Prove that there exists a real number λ such that ∇g (p) = λ ∇f (p)

18. (a) Prove that a parameterized curve α: I→S is a geodesic iff its covariant derivative (α̇)' is zero along α

(b) Let S be an n-surface in R^{n+1}, let p, q ∈ S ,and let α be a piecewise smooth parameterized curve from p to q. Prove that the parallel transport $P_\alpha : S_p \to S_q$ along α is a vector space isomorphism which preserves dot product

19. (a) Prove that the Weingarten map Lp is self- ad joint

(b) For each 1-form ω on U (U open in R^{n+1}) prove that there exists unique functions f_i : U→R ($i \in \{1,2, \ldots \ldots . n+1\}$) such that $\omega = \sum_{i=1}^{n+1} f_i \, d x_i$. Also show that ω is smooth iff each f_i is smooth

20. (a) Find the Gaussian curvature of $\phi(t, \theta) = (cos\theta, sin\theta, t)$

(b) Prove that on each compact oriented n surface S in R^{n+1}there exists a point p such that the second fundamental form is definite

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(10 x 4 = 40)