Reg. No.. Name...

M.SC DEGREE END SEMESTER EXAMINATION OCTOBER 2016

SEMESTER **– 3: MATHEMATICS**

COURSE: **P3MATT12 – FUNCTIONAL ANALYSIS**

Common for Regular (2015 Admission) & Supplementary / Improvement (2014 Admission)

Time: Three Hours Max. Marks: 75

PART A

(Answer **any five** of the following. Each question carries 2 marks)

1. Define the operations vector addition and scalar multiplication in C$[a,b]$.
2. Prove that every linear operator T: D(T) $\rightarrow$Y takes a linearly independent set in D(T) in to a linearly independent set in Y.
3. The operator T : $R^3 \rightarrow R^3$ is defined by T(ξ_1, ξ_2, ξ_3) = (ξ_1, ξ_2, $-\xi_1 - \xi_2$). Find the R (T) and N(T).
4. State the Apollonius Identity for inner product spaces.
5. Show that the annihilator M$^\perp$ of a set M ≠ Φ in an inner product space X is closed subset of X.
6. Two Hilbert spaces H and $\check{H}$ both real or complex are isomorphic then prove that they have the same Hilbert dimension.
7. Prove that in every Hilbert space $H \neq \{0\}$, there exist a total orthonormal set.
8. Prove that the canonical map C: X$\rightarrow$ X'' given by C(x) = g_x (x ε X) is an isomorphism of X $on¿$ X''.

(5 x 2 =10)

PART B

(Answer **any five** of the following. Each question carries 5 marks)

9. a) Prove that every finite dimensional subspace Y of a normed space X is complete.

   b) Prove that on a finite dimensional vectorspace any norm $||.||$ is equivalent to any other norm $||.||_0$
10. Prove that the space C $[a,b]$is a Banach space.
11. a) Define the compactness of a metric space X.

   b) Prove that a compact subset M of a metric space is closed and bounded. Also prove that the converse of this is generally false.
12. Define B(X,Y). Show that the dual X' of a normed space X is a Banach space.

13. a) Prove that not all normed spaces are inner product spaces.

b) Show that the Unitary space C^n is a Hilbert space.

14. Explain Gram-Schmidt Process for orthonormalizing a linearly independent sequence.

15. a) If a Hilbert space H is seperable then prove that every orthonormal set in H is countable.

b) Define a sesquilinear form and prove that the inner product < , > is a sesquilinear form.

16. a) Prove that every finite dimensional normed space is reflexive.

b) Define the terms meager and nonmeager of a metric space

c) State Baire's Category theorem.

(5 x 5 = 25)

PART C

(All questions below are compulsory. Answer either **(A) or (B).** Each question carries 10 marks)

17. (A) a) State and prove Riesz lemma.

b) Prove that every linear operator on a finite dimensional normed space is bounded.

(B) State and prove the completion theorem of normed spaces

18. (A) Show that the dual of l^2 is l^2

(B) a) state and prove the parallelogram equality.

b) If in an inner product space $x_n \to x$ and $y_n \to y$ then prove that $< x_n , y_n > \to < x , y>$

c) Define the isomorphisms of inner product spaces

19. (A) a) When a sesquilinear functional is called bilinear? Define the norm of a sesquilinear functional.

b) State and prove Reisz Representation theorem (Sesquilinear form on Hilbert spaces)

(B) a) Define a Hilbert adjoint operator with an example.

b) Let the operator T:$H_1 \to H_2$ be bounded, where H_1&$H_2$are Hilbert spaces. Then, prove that its Hilbert adjoint operator $T^{¿}$ exists, is unique and is a bounded linear operator with norm || = ||' ¿ || T||.

20. (A) a) State and prove the Uniform boundedness theorem.

b) Write an application of Uniform boundedness theorem

(B) Explain the relation between the adjoint operator $T^{\times}$ and the Hilbert adjoint operator $T^{¿}$

(4 x 10 = 40)

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