Reg. No... Name...

M.SC DEGREE END SEMESTER EXAMINATION OCTOBER 2016

SEMESTER – 3: MATHEMATICS

COURSE: P3MAT11– MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

Common for Regular (2015 Admission) & Supplementary / Improvement (2014 Admission)

Time: Three Hours Max. Marks: 75

PART A

(Answer **FIVE** questions. Each carries 2 marks.)

1. Define convolution of two real valued function f and g.
2. Define Fourier series of a periodic function f with period p.
3. Define the directional derivative of a function f at c in the direction u.
4. What do you mean by the Jacobian matrix?
5. If f = u + iv is a complex valued function with a derivative at a point z in C, then J_f(z) = lf'(z)llf'(z)l.
6. State mean value theorem for differentiable functions.
7. What do you meant by primitive mappings?
8. Define Stokes theorem. (2 x 5 = 10)

PART B

(Answer **FIVE** questions. Each carries 5 marks)

9. Use the Fourier integral theorem to evaluate the improper integral

$$\frac{2}{\pi}\int_0^{\infty}\frac{\sin v\cos vx}{v}\,dv$$

10. State and prove Weierstrass approximation theorem.
11. Let u and v be two real valued functions defined on a subset S of the complex plane. Assume also that u and v are differentiable at an interior point c of S and that the partial derivatives satisfy the Cauchy-Riemann equations at c. Then the function f = u + iv has a derivative at c. Moreover, f'(c) = D_1u(c) + i D_1v(c).
12. Compute the gradient vector ∇f(x,y) at those points (x,y) in R^2 where it exists:

    f (x,y) = x^2y^2 log(x^2+y^2) if (x, y) ≠ (0, 0)
    = 0 if (x, y) ¿ (0, 0)
13. Consider the function f(x, y) = xy (x^2-y^2)/(x^2+y^2)) if (x, y) ≠ (0, 0)= 0 if (x, y) ¿ (0, 0) Show that $D_{1,2}$ f(x,y) ≠ $D_{2,1}$ f(x,y).
14. Find and classify the extreme values (if any) of the function f(x,y) = y^2 - x^3.
15. If ω and $\lambda$are k- and m- forms, respectively, of class ϱ in E , then

    d(ω¿ λ¿=¿) ^ $\lambda+(-1)^k$ ω¿ (dλ¿.
16. For every f ∈ϱ¿), show that L (f) = L'(f). (5 x 5 = 25)

PART C

(Answer **ALL** questions. Each carries 10 marks.)

17. A. Let R = (- ∞,+∞¿.Assume that f, g ∈ L(R), and that atleast one of f or g is continuous and bounded on R.Let h denote the convolution, h = f*g. Then for every real u we have

$$\int_{-\infty}^{+\infty} h(x)e^{-ixu}\,dx = \left(\int_{-\infty}^{+\infty} f(t)e^{-itu}dt\right)\left(\int_{-\infty}^{+\infty} g(y)e^{-iyu}dy\right).$$

The integral on the left exists both as a Lebesque integral and as an improper Riemann integral.

OR

17. B. Let f : $R^2 \rightarrow R^3$ be defined by the equation

f(x, y) = (sinx cosy, sinx siny, cosx cosy). Determine the Jacobian matrix D f(x,y)

18. A. Assume that g is differentiable at a, with total derivative g'(a). Let b = g(a) and assume that f is differentiable at b, with total derivative f'(b). Then prove that the composite function h = f ∘ g is differentiable at a, and the total derivative h'(a) is given by

h'(a) = f'(b) ∘ g'(a), the composition of the linear functions f'(b) and g'(a).

OR

18. B. a) If x(r, $\theta ¿= r\,cos\theta, y(r,\theta) = r\,sin\theta$, show that $\frac{\partial(x,y)}{\partial(r,\theta)}$ = r.

b) If If x(r, $\theta,\varnothing ¿= r\,cos\theta sin\varnothing, y(r,\theta,\varnothing) = r\,sin\theta sin\varnothing$, z $(r,\theta,\varnothing) = r\cos\varnothing$, show that

$\frac{\partial(x,y,z)}{\partial(r,\theta,\varnothing)}$ = -r^2sin∅.

19. A. Prove that if both partial derivatives $D_r f$ and $D_k f$ exists in an n-ball (c, δ¿ and if both are differential at c. Then $D_{r,k}$ f(c) = $D_{k,r}$ f(c).

OR

19. B.For each of the following functions verify that the mixed partial derivatives $D_{1,2}$ f and $D_{2,1}$ f are equal a) f(x,y) = tan (x^2/y), y≠ 0

b) f(x,y) = $x^4+y^4 - 4x^2y^2$ (x,y) $\neq (0,0)$

20. A. Suppose F is a C-Mapping of an open subset E in R^n into R^n , 0∈ E, F(0) = 0, and F'(0) is invertible. Then prove that there is a neighborhood of 0 in R^n in which a representation F(x) = B_1 B_2......B_{n-1} G_n .....G_1 (x) is valid, where each G_i is a primitive C – Mapping in some neighborhood of 0. G_1 (0) = 0, G_1'(0) is invertible, and each B_i is either a flip or the identity operator.

OR

20.B. For (x, y) $\in R^2$, Define F(x, y) = (e^xcosy – 1, e^xsiny)

Prove that F = $G_1 \circ G_2$, where $G_1(x,y) = ¿$cosy – 1, y) $G_2(u,v) = (u,(1+u)tanv)$ are primitive in some neighbourhood of (0,0).

*********** (10 x 4 = 40)