Reg. No...
Name..

M.SC. DEGREE END SEMESTER EXAMINATION NOVEMBER 2016

SEMESTER **– 1: MATHEMATICS**

COURSE: **16P1MATT05 –: COMPLEX ANALYSIS**

Time: Three Hours

Max. Marks: 75

PART A

(Answer **all** Questions. Each question carries 1.5 Marks

1. Show that an analytic function defined in a region Ω reduces to a constant if its argument is constant.
2. Find the fixed points of the linear transformation w = $\frac{3z-4}{z-1}$.
3. Prove that the linear transformation carries circles into circles.
4. Compute $\int_{|z|=r} x\,dz$ for the positive sense of the circle.
5. Let γ be a piecewise smooth differentiable closed curve, show that n($\gamma, a¿$ = 0 in the unbounded region.
6. Compute $\int_{|z|=2} z^n (1-z)^{-m} dz$.
7. Define: Poles, Removable singularity, Essential singularity .
8. Prove that if f(z) is analytic and nonconstant in a region Ω then its absolute value $|f(z)|$ has no maximum in Ω.
9. Find the poles and residues of the function $\frac{1}{(z^2-1)^2}$.
10. Show that any harmonic function which depends only on r must be of the form a $\log r + b$.

(1.5 x 10 = 15)

PART B

(Answer **any FOUR** Questions. Each question carries 5 Marks)

11. Show that the transformation w = $\frac{az+b}{cz+d}$ is the resultant of three transformations of the form w = z + , w = kz and w = $\frac{1}{z}$ where α and k are complex constants.
12. Prove that if a linear transformation carries a circle C_1 into a circle C_2 then it transforms any pair of symmetric points with respect to C_1 into a pair of symmetric points with respect to C_2 .

13. Prove that let f(z) be analytic on the rectangle R' obtained from a rectangle R by omitting a finite number of interior points ζ_j. If it is true that $\lim_{z \to \zeta_j} ¿¿$) f(z) = 0 ,for all j , then $\int_{\partial R} f(z) dz = 0$.

14. State and prove Schwarz lemma.

15. State and prove Cauchy's residue theorem.

16. Prove that the arithmetic mean of a harmonic function over concentric circles $|z| = r$ is a linear function of log r , $\frac{1}{2\pi} \int_{|z|=r} u \, d\theta = \alpha \log r + \beta$.

(5 x 4 = 20)

PART C

(Answer **either (a) or (b**) of the following **FOUR** Questions. Each question carries 10 Marks)

17. (a) Describe the geometrical interpretation of symmetry.

OR

(b) Prove that ($z_1 z_2 z_3 z_4$¿ is real if and only if the four points lie on a circle or on a straight line.

18. (a) (1) State and prove Cauchy's theorem in a disc.

(2)State and prove Cauchy's integral formula.

OR

(b) Suppose $\varphi(\tau)$ is continous on an arc γ, then show that the function

F_n(z) = $\int_{\gamma} \frac{\varphi(\tau)}{(\tau - z)^n} d\tau$ is analytic in each of the region determined by γ

and $F_n'(z) = n F_{n+1}(z)$.

19. (a) Prove that if f(z) is analytic in Ω then $\int_{\gamma} f(z) dz = 0$ for every cycle γ which is homologus to zero in Ω.

OR

(b) State and prove Taylor's theorem.

20. (a) Evaluate by the method of residues $\int_0^{\infty} \frac{\cos x}{x^2 + a^2} dx$,a is real.

OR

(b) State and prove Poisson's formula.

(10 x 4 = 40)

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