Reg. No…………………………………………… Name…………………………………

M. Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER 2016

SEMESTER **– 1: MATHEMATICS**

COURSE: **P1MATT04–: GRAPH THEORY**

(Supplementary / Improvement for 2015 Admission)

Time: Three Hours Max. Marks: 75

PART A

Answer **any five** questions. Each question carries 2 marks

1. Define a self-complementary graph with an example.
2. Define a line graph of a graph. What is the line graph of $K_{1,4}$
3. Define the diameter and centre of a graph.
4. Prove that if $\delta(G) \geq 2$, then G contains a cycle.
5. For every graph G with n vertices, prove that $\alpha + \beta = n$.
6. Prove that if $\chi(G) = 2$ then G is a bipartite graph with atleat one edge
7. Define dual of a plane graph with an example.
8. Prove that for a simple graph G, $\Delta(G) \leq \chi'(G)$. (2 x 5 = 10)

PART B

Answer **any five** questions. Each question carries 5 marks

9. Prove that every tournament contains a directed Hamiltonian path.
10. Prove that for any loop less connected graph, $\kappa(G) \leq \lambda(G) \leq \delta(G)$. Also draw a graph with $\kappa = \lambda = \delta$.
11. Prove that a connected graph G with atleast two vertices is a tree if and only if, its degree sequence ¿) satisfies the condition : $\sum_{i=1}^{n} d_i = 2(n-1)$ with $d_i > 0$ f each i.
12. Explain Prim's Algorithm with an example.
13. Prove that if for a simple 2- connected graph G $\leq \kappa$, then G is Hamiltonian.
14. Prove that for every positive integer k, there exist a triangle free graph with chromatic number k.
15. Is $K_{3,3}$ planar? Justify.
16. State and prove Euler's Formula

(5 x 5 = 25)

PART C

Answer **either Part I or Part II** of each question. Each question carries 10 marks

17. (I) (a) Prove that the set of all automorphisms of a simple graph G is a group with respect to the composition of mappings as the group operation

(b) Prove that a graph is bipartite if, and only if, it contains no odd cycles

(II) (a) Prove that the line graphs of two isomorphic simple graphs are isomorphic

(b) State and Prove Moon's theorem.

18. (I) State and Prove Cayley's theorem

(II) Explain Dijikstra's Algorithm with an example.

19. (I) (a) Let G be a simple graph with $n \geq 3$vertices. Prove that if for every pair of non adjacent vertices u, v of G, $d(u)+d(v) \geq n$, then G is Hamiltonian.

(b) Define closure of a graph with an example. Also prove that the closure of a graph is well defined.

(II) State and Prove Brook's theorem

20. (I) State and prove Vizings theorem

(II) (a) State and prove Heawood five color theorem

(b) If G is a simple planar graph with at least three vertices, then $m \leq 3n-6$

(10 x 4 = 40)

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