B.Sc. DEGREE END SEMESTER EXAMINATION OCTOBER 2016
SEMESTER- 1: MATHS COMPLEMENTARY FOR PHYSICS/CHEMISTRY
COURSE- U1CPMAT1: DIFFERENTIAL CALCULUS AND TRIGONOMETRY

For Supplementary (2014 Admission)

Time: Three Hours   Max Marks: 75

Part A

Short Answer Questions. *Answer **all** questions. Each question carries **one** mark.*

1. Find the value of $\lim\limits_{\theta \to 0}\left(3+\dfrac{\sin 3\theta}{3\theta}\right)$.

2. If $3-x^3 \leq g(x) \leq 3\sec x$ for all x, find $\lim\limits_{x \to 0} g(x)$.

3. If $f(x)=(x-1)^2+2$, find $f'(0)$.

4. Is the function f(x) = x^2 continuous at x = 0 ?

5. Functions with the same derivative differ by a _____.

6. Find the value of f(x, y, z) = $\sqrt{x^2+y^2-z^2}$ at (4,-3,0).

7. Find the value of $\dfrac{\partial f}{\partial x}$ at (0,1) where f(x, y) = $\dfrac{x}{y^2}$.

8. The period of cos x is _____.

9. Write the relation connecting circular tan function and hyperbolic tan function.

10. Prove that $e^{ix}=\cos x+i\sin x$.

(1 × 10 = 10)

Part B

Brief Answer Questions. *Answer **any eight** questions. Each question carries **two** marks.*

11. Let f(x) = $\begin{cases} 3-x, & x<2 \\ \dfrac{x}{2}+1, & x>2 \end{cases}$

    (a) Find $\lim\limits_{x \to 2^+} f(x)$ and $\lim\limits_{x \to 2^-} f(x)$

    (b) Does $\lim\limits_{x \to 2} f(x)$ exist? Justify.

12. Find the parametrization of the line segment with end points (-1,-3) and (4,1).

13. If y = sin u and u = 3x + 1 then find $\dfrac{dy}{dx}$.

14. If $x^3 + y^3 = 18xy$, find $\frac{dy}{dx}$ using implicit differentiation.

15. Find the function g(x) whose derivative is $\frac{1}{x^2} + 2x$ and passing through (-1,1).

16. If f(x,y) = ye^x, find the first order partial derivatives.

17. Find $\frac{dw}{d\theta}$ at $\theta = \frac{\pi}{2}$ using chain rule, where w = xy, x=cos θ and y = sin θ

18. If $x = \cos\theta + i\sin\theta$, find $x^n - \frac{1}{x^n}$

19. Prove that $\cosh^2 x - \sinh^2 x = 1$.

20. Write the infinite series expansion of coshx.

$$(2 \times 8 = 16)$$

Part C

Short Essay Type Questions. Answer **any five** questions. Each question carries **5** marks.

21. Find a δ > 0, to show that $\lim_{x \to 10} \sqrt{19-x}$ = 3 that works for ε = 1

22. Find the equation of the tangent and the normal to the curve $x^2 - y^2 + xy = 1$ at (2,3)

23. State and prove Mean Value Theorem.

24. Find all the first and second partial derivatives of $f(x,y) = x + y + xy$.

25. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s ,if

$w = x + 2y + z^2, x = \frac{r}{s}, y = r^2 + \ln s, z = 2r$

26. Expand cos^{7}θ in a series of cosines of multiples of θ.

27. Separate $\tan(\alpha + i\beta)$ into real and imaginary parts.

$$(5 \times 5 = 25)$$

Part D

Essay. Answer any **two** questions. Each question carries **12** marks.

28. a) Find the derivative of $y=\sqrt{x}$ for x > 0. Find the tangent line to the curve at x = 4.

b) Prove that if f has a derivative at x = c, then f is continuous at x= c.
Is the converse true? Give an example.

29. Find the critical points of $f(x) = x^{4/3} - 4x^{1/3}$. Identify the intervals on which f is increasing and
decreasing. Find the local and extreme values of the function.

30. a) $z=\tan^{-1}\dfrac{y}{x}$, prove that $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial y^2}=0$.

b) Prove the mixed derivative theorem $f_{xy}= f_{yx}$ for f(x, y) = xcosy + ye^x .

31. Sum the series $\dfrac{1}{2}\sin\alpha+\dfrac{1}{2}\cdot\dfrac{3}{4}\sin 2\alpha+\dfrac{1}{2}\dfrac{3}{4}\dfrac{5}{6}\sin 3\alpha+....$

(12 × 2 = 24)

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