P1644

Reg. No………………………………Name………………………………..

M. Sc. DEGREE END SEMESTER EXAMINATION NOVEMBER 2016

SEMESTER – 1: MATHEMATICS

COURSE: 16P1MATT04 –: ORDINARY DIFFERENTIAL EQUATIONS

Time: Three Hours Max. Marks: 75

PART A

(Each Question has 1.5 Marks Answer **All** Questions)

1. Find any one characteristic vector of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$.
2. Show that $(3e^{7t}, 2e^{7t})$ is a solution of the system $\frac{dx}{dt} = 5x + 3y$, $\frac{dy}{dt} = 4x + y$.
3. Find the derivative of the vector function $\emptyset(t) = \begin{pmatrix} 5t^2 \\ -6t^3 + t^2 \\ 2t^2 - 5t \end{pmatrix}$.
4. Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{n^3 \ (x+5)^n}{6^n}$
5. Solve using Picard's method : $y' = x + y,\ y(0) = 1.$
6. Define Sturm-Lioville Problem.
7. Show that the function $sinx$ and $sin2x$ are orthogonal with respect to the weight function having the constant value 1 on the interval $0 \leq x \leq \pi$.
8. Find a Laplace transform of $\sinh ax$.
9. Evaluate $\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x}\, dx$.
10. Define the convolution of the functions (x) and $g(x)$.

(1.5 x 10 = 15)

PART B

Each Question has 5 marks Answer **any Four**

11. Find the general solution of the linear system $\frac{dx}{dt} = 6x - 3y,\ \frac{dy}{dt} = 2x + y$.
12. Show that the set of vector functions

$$\emptyset_1(t) = \begin{pmatrix} e^{2t} \\ 2e^{2t} \\ 5e^{2t} \end{pmatrix}, \emptyset_2(t) = \begin{pmatrix} e^{2t} \\ 4e^{2t} \\ 11e^{2t} \end{pmatrix}, \emptyset_3(t) = \begin{pmatrix} e^{2t} \\ e^{2t} \\ 2e^{2t} \end{pmatrix}$$

defined for all t is linearly dependent on any interval $a \leq t \leq b$.

13. The equation $4x^2y'' - 8x^2y' + (4x^2 + 1)y = 0$ has only one Frobenius series solution. Find the general solution.
14. Find the solution of Sturm- Liouville problem $y'' + \lambda\, y = 0$, $y(0) = 0,\ y(\pi) = 0$ when ≥ 0.
15. Find $L^{-1}\left[\frac{1}{(p^2+a^2)^2}\right]$ by convolution.
16. Find the Laplace transform of x^n.

(5 x 4 = 20)

PART C

Each Question has 10 Marks Answer (**A) or (B**) from Each Questions

17. (A) Find the characteristic values and characteristic vectors of the matrix
$\begin{pmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ -2 & 1 & -1 \end{pmatrix}$.

(B) Show that x$= 2t + 1, y = -$ is a particular solution of the non homogeneous linear system $\frac{dx}{dt} = 2x - y - 5t$, $\frac{dy}{dt} = 3x + 6y - 4$. Also find the general solution of this system.

18. (A) Show that the Frobenius series solution of $x^2y'' + xy' + x^2y = 0$ is $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} x^{2n}$.

(B) Find the general solution of $y'' + xy' + y = 0$ and verify that the two series of solutions converge for all x.

19. (A) Find the characteristic values and characteristic functions of the Sturm-Liouville problem

$$\frac{d}{dx}\left\{x \frac{dy}{dx}\right\} + \frac{\lambda}{x} y = 0, \ y'(1) = 0, y'(e^{2\pi}) = 0$$ where ≥ 0.

(B) Obtain the formal expansion of the function $f(x) = \pi x - x^2, 0 \leq x \leq \pi$ in the series of orthonormal characteristic functions $\{\emptyset_n\}$ of the Sturm-Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0$,

$y(0) = 0, \ y(\pi) = 0$.

20. (A) (i) Find $L\left(f(t)\right) = \frac{p+7}{p^2+4p+8}$ for $f(t)$.

(ii) Evaluate $\int_0^{\infty} \frac{\sin xt}{t} dt$.

(B) (i) Find the Laplace transform of $x^{\frac{-1}{2}}$.

(ii) Use Laplace transform to solve $y'' + 4y = 4x, \ y(0) = 1, \ y'(0) = 5$.

(10 x 4 = 40)

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