Reg. No…………………………………….   Name………………………………

M. Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER 2016

SEMESTER **– 1: MATHEMATICS**

COURSE: **16P1MATT03 –: MEASURE THEORY AND INTEGRATION**

Time: Three Hours   Max. Marks: 75

PART A

(Answer **all questions**. Each question carries 1.5 marks *)*

1. Prove that the Lebesgue outer measure m* is translation invariant.

2. Show that if E_1 and E_2 are measurable, then $m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2$.

3. Show that if f is a measurable function and f=g a.e., then g is measurable.

4. Let $\phi = \sum_{i=1}^{n} a_i \chi_{E_i}$, with $E_i \cap E_j \neq \varphi$ for $i \neq j$. Suppose each E_i is a measurable set of finite measure. Then show that $\int \phi = \sum_{i=1}^{n} a_i m E_i$.

5. If f and g are bounded measurable functions defined on a set *E* of finite measure then show that $\int_E af + bg = a\int_E f + b\int_E g$

6. Let *f* be a non- negative measurable function. Show that $\int f = 0$ implies $f = 0 \ a.e.$

7. Define Measure Space. Give an example.

8. If $E_i \in$ B, then show that $\mu\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} \mu\left(E_i\right)$.

9. Show that countable union of positive sets is positive.

10. If Y is any class of subsets of X, then show that there exist a smallest monotone class $M_0(Y)$ containing Y.

(1.5 x 10 = 15)

PART B

Answer **any four** of the following .Each question carries 5 marks

11. Show that the interval (a, ∞) is measurable.

12. Define Cantor set **P**. Show that **P** is measurable.

13. Let f be non negative function which is integrable over a set *E*. Then show that

given ϵ>0, there is δ>0 such that for every set *A* subset of *E* with $m(A)<\delta$, $\int_A f<¿$.

14. State and prove Lebesgue Convergence Theorem.

15. Let μ be a measure defined on a σ-algebra **a**. Then show that the set function μ*

defined by μ*(E)= $\inf \sum_{i=1}^{\infty} A_i$, A_i ϵ**a**.and $E \subset ¿_{i=1}^{\infty} A_i$ is an outer measure on a..

16. Show that if E ϵ S XT, then for each x ϵ X and *y* ϵ Y, E_x ϵ T and E^y ϵ S.

(5 x 4 = 20)

PART C

*Answer **either** **I** **or** **II** of each question below. Each question carries 10 marks*

17. I. a).Show that outer measaure of an interval is its length.

b).Define algebra. Show that the family M of measurable sets is an algebra.

OR

II. a). Show that there exist a set which is not measurable.

b). Show that a set A is measurable if and only if its characteristic function is measurable.

18. I. a). Let *f* be defined and bounded on a measurable set E with *mE* finite. Show that

$$\inf_{f \le \psi} \int_E \psi(x)\,dx = \sup_{f \ge \phi} \int_E \phi(x)\,dx$$

in order for all simple functions φ and ψ, it is necessary and sufficient that *f* is measurable.

b). Let f be a bounded function defined on [a,b].Show that f is Reimann integrable on [a,b], then it is measurable and $R\int_a^b f(x)\,dx = \int_a^b f(x)\,dx.$

OR

II. a). State and prove Fatou's lemma.

b). State and prove Monotone Convergence theorem.

19\. I. a) . Let E be a measurable set such that $0< \nu(E)< \infty$. Then show that there is a positive

set A contained in E with $\nu(A)>0$.

b). State and prove Hahn Decomposition Theorem.

OR

II. State and Prove Caratheodory's Theorem.

20\. I. Let [[X, S,μ]] and [[Y,T, ν]] be σ- finite measure spaces. For V ϵ SXT write

$\varphi(x)=\nu(V_x)$, $\psi(y)=\mu(V^y)$, for each x ϵX and y ϵY. Then show that φ is S-measurable, ψ is

T- measurable and $\int_X \varphi \, dx = \int_Y \psi \, dv$.

OR

II. (a). Show that the class of elementary sets **E** is an algebra.

(b). Let f be a non negative S XT- measurable function and

let $\varphi(x)=\int_Y f_x dv, \ \psi(y)=\int_X f^y d\mu$

for each xϵX, yϵY; then show that φ is S- measurable, ψ is T-measurable and

$$\int_X \varphi d\mu = \int_{XxY} fd(\mu X v) = \int_Y \psi dv$$.

(10 x 4 = 40)

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