Name.......................................Reg. No.......................................

M. Sc. DEGREE END SEMESTER EXAMINATION - NOVEMBER - 2016

SEMESTER **– 1: MATHEMATICS**

COURSE: **P1MATT02 – BASIC TOPOLOGY**

(For Supplementary / Improvement 2015 Admission)

Time: Three Hours | Max. Marks: 75

PART A

(Answer **any 5** questions. Each carries 2 marks.)

1. Prove that A is closed in X if and only if $\acute{A} = A$.
2. Define a metrisable space. Give an example of a space which is not metrisable.
3. Prove or disprove: every continuous image of a compact space is compact.
4. Give an example of a separable space which is not second countable.
5. Is the set of rational numbers connected? Justify.
6. Prove that the components are closed sets.
7. Define (i) Path connected
   (ii) Locally connected
8. Prove or disprove: a compact subset in a Hausdorff space is open.

(2 x 5 = 10)

PART B

(Answer **any 5** questions. Each carries 5 marks)

9. Let X be a set and **B** a family of its subsets covering X. Prove that for any $B_1, B_2 \in$ **B** and $x \in B_1 \cap B_2$,there exists $B_3 \in$ **B** such that $x \in B_3$ and $B_3 \subset B_1 \cap B_2$ if and only if there exists a topology on X with **B** as base .
10. Define neighbourhood and show that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.
11. Prove that the every open, surjective map is a quotient map.
12. Prove that every second countable space is Lindeloff.
13. Prove that if X is locally connected then components of open subsets of X are open.
14. Prove that every continuous real valued function on a compact space is bounded.
15. Prove that regularity is a hereditary property.
16. Prove that every compact Hausdorff space is T_3.

(5 x 5 = 25)

PART C

(Answer either (a) or (b) of the following 4 questions. Each carries 10 marks.)

17. (a) Prove that if a space is second countable then every open cover of it has a countable subcover.

(b) Prove that a subset A of a space X is dense in X if and only if for every non empty open subset B of X, $A \cap B \neq \varnothing$.

18. (a) State and prove Lebesgue covering lemma.

(b) Prove that every second countable space is first countable.

19. (a) Prove that every quotient space of a locally connected space is locally connected.

(b) Prove that every closed and bounded interval is compact.

20. (a) Define Tychonoff space and show that every Tychonoff space is T_3.

(b) If A ,B be compact subsets of topological spaces X,Y respectively and W be an open subset of $X \times Y$ containing the rectangle $A \times B$, then prove that there exist open sets U,V in X,Y respectively such that $A \subset U, B \subset V \wedge U \times V \subset W$.

(10 x 4 = 40)

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