Reg. No……………………………………Name……………………………………………

M.Sc. DEGREE END SEMESTER EXAMINATION OCTOBER – NOVEMBER 2016

SEMESTER – 1 : MATHEMATICS

COURSE P1MATT01: LINEAR ALGEBRA

(For Supplementary / Improvement - 2015 Admission)

Time: Three Hours Max. Marks: 75

Part A

(Answer any **five** questions. Each question carries 2 marks)

1. Find the span of the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ in $\mathbf{R}^3$.

2. Find the range, rank and null space of the zero transformation.

3. What you mean by a non-singular linear transformation. Show that the transformation

   $T(x_1, x_2)=(x_1+x_2, x_1)$ is non-singular..

4. Let V be the space of all polynomial functions from **R** into **R** of the form $f(x) = C_0+C_1x+C_2x^2+C_3x^3$.

   Find $[D]_B$, where D is the differentiation operator and $B =\{1, x, x^2, x^3\}$.

5. Let A be a 2 x 2 matrix over K. Show that (adjA)A =A(adjA)=(detA)I.

6. Define (i) Characteristic value (ii)Characteristic vector and (iii) characteristic space.

7. If det denote the unique determinant function on 2 x 2 matrices over K, show that

   det(adj A)=det(A).

8. Find an invertible real matrix P such that $P^{-1}AP$ and $P^{-1}BP$ are both diagonal where

   $A=\begin{bmatrix}1 & 2\\ 0 & 2\end{bmatrix}, B=\begin{bmatrix}3 & -8\\ 0 & -1\end{bmatrix}$.

(2 x 5 = 10)

Part B

Answer any **five** questions. Each question carries 5 marks

9. Let V be the vector space of all functions from **R** into **R**. Let V_e be the subset of even functions,

   $f(-x)=f(x)$, let V_o be the subset of odd functions ,$f(-x)= -f(x)$

   (i). Prove that V_e and V_o are subspaces of V .

   (ii). Prove that $V_e + V_o = V$

   (iii).Prove that $V_e \cap V_o = \varphi$

10. If A is an m x n matrix with entries in the field F, then show that row rank(A)=column rank(A).

11. Let V be a finite dimensional vector space over the field F and let B = {α_1 ,α_2.... α_n} be a basis for V. Then show that there is a unique dual basis B* = {f_1 ,f_2.... f_n} for V* such that $f_i(\alpha_j)=\delta_{ij}$. Also show that for each linear functional f on V,

$$f=\sum_{i=1}^{n} f(\alpha_i) f_i$$ and for each α in V, $$\alpha=\sum_{i=1}^{n} f_i(\alpha)\alpha_i$$.

12. Let V and W be finite dimensional vector spaces over the field F. Let B be the ordered basis for V with dual basis B*and let B' be an ordered basis for W with dual basis B'*. Let T be a linear transformation from V into W. Let A be the matrix of T relative to B, B' and let B be the matrix of T^t relative to B'*, B *. Show that $B_{ij} = A_{ji}$.

13. Use Cramer's rule to solve the system of linear equation over the field of rational numbers.

x+y+z =11
2x-6y-z=0
3x+4y+2z=0.

14. Let T be a linear operator on $\mathbf{R}^3$ which is represented in the standard ordered basis by the matrix

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$. Prove that T is diagonalizable by exhibiting a basis for $\mathbf{R}^3$, each vector of which is a characteristic vector of T.

15. Let W be an invariant subspace for T. Show that the characteristic polynomial for the restriction operator T_W divides the characteristic polynomial for T. Show also that the minimal polynomial for T_W divides the minimal polynomial for T.

16. Let F be a commutating family of diagonalizable linear operators on the finite dimensional vector space V. Show that there exist an ordered basis for V such that every operator in F is represented in that basis by a diagonal matrix.

(5 x 5 = 25)

Part C

Answer part 1 or part II of each questions. Each question carries 10 marks

17.I.(a). Let B = {α_1, α_2, α_3} be an ordered basis for **R**^{3} consisting of α_1={1, 0, -1}, α_2={1, 1, 1}, α_3= {1, 0, 0}.

What are the coordinates of the vector (a, b, c) in the ordered basis B.

(b). Let m and n be positive integers and let F be a field. Suppose W is a subspace of F^n and dim W ≤ m. Show that there is precisely one m x n row reduced echelon matrix over F which has W as its subspace.

II.(a). Let V be a vector space over the field F of complex numbers. Suppose α, β and γ are linearly independent vectors in V. Prove that (α+ β), (β + γ)and (α+ γ) are linearly independent.

(b). Let R be a non-zero row-reduced echelon matrix. Show that the non-zero vectors of **R** form a basis for the row space of R.

18. I. (a). Show that the set of all invertible operators on a vector space V together with the operation function composition is a group.

(b). Let V be a finite dimensional vector space and let T be a linear operator on V. Suppose that rank(T)= rank(T^2). Prove that the range and null space of T are disjoint. ie, they have only the zero vector in common.

II. (a). Let V be a finite dimensional vector space over the field F, and let W be a subspace of V. Show that dim W +dim W^0 =dim V.

(b). If f is a nonzero linear functional on a vector space V, show that the null space of f is a hyperspace in V. Conversely show that every hyperspace in V is a null space of a (not unique) nonzero linear functional on V.

19. I. (a). Let D be an n-linear function on n x n matrices over K. Suppose D has the property that

D(A)=0 whenever two adjacent rows of A are equal. Show that D is alternating.

(b). Show that a linear combination of n-linear functions is n-linear.

II. (a). Let T and U be linear operators on the finite dimensional vector space V. Prove that

(i). det(TU) = (det T)(det U)

(ii). Define orthogonal matrix. If *A* is orthogonal, show that. Give an example of an orthogonal matrix for which det *A* = -1.

(b). If A is an invertible n x n matrix over a field F, show that det A $\neq$ 0

20. I. State and prove Cayley- Hamilton Theorem.

II. (a). Let T be a linear operator on the finite- dimensional space V. Let $c_1, \ldots, c_k$ be the distinct characteristic values of T and let W_i be the span of characteristic vectors associated with the characteristic value c_i. If $W = W_1 + \ldots + W_k$, then show that dim W = dim W_1 +.........+dim W_k. If B_i is an ordered basis for W_i, then show that $B = (B_1, \ldots, B_k)$ is an ordered basis for W.

(b). Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Show that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.

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