

MSc DEGREE END SEMESTER EXAMINATION – NOVEMBER 2015

SEMESTER: 1: SUBJECT – PHYSICS

COURSE: P1PHYT01 -MATHEMATICAL METHODS IN PHYSICS – I

(Regular, Supplementary / Improvement)

Time: Three Hours

Max. Marks: 75

Part A (Objective Type)

(Answer all questions. Each question carries 1 Mark)

- The condition for a solenoidal vector field is
(a) $\text{Grad } A = 0$ (b) $\text{Div } A = 0$ (c) $\text{Curl } A = 0$ (d) $\text{Curl } A = \text{finite}$
- The Eigen values of the matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ are
(a) $\pm e^{i\theta}$ (b) $e^{\pm i\theta}$ (c) $\cos\theta \pm \sin\theta$ (d) $\pm \tan\theta$
- If $AY = PY$, then $Y =$
(a) PYA (b) PYA^{-1} (c) $A^{-1}PY$ (d) PYP^{-1}
- The value of ϵ_{ppq} - tensor is
(a) 1 (b) -1 (c) 0 (d) infinity
- The value of $\Gamma^{1/2}$ is
(a) 0 (b) $\pi/2$ (c) π (d) $\sqrt{\pi}$

(1 x 5 = 5)

Part B (Short answer)

(Answer any 5 questions. Each question carries 2 Marks)

- Explain Gauss's divergence theorem
- Express the position and velocity vectors of a particle in cylindrical coordinates
- Prove that the Eigen values of a Hermitian matrix are always real
- Evaluate a, b and c when $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal
- Define covariant, contravariant and mixed tensors. Give examples.
- Show that the Kronecker delta is a mixed tensor of rank 2.
- Using the generating function for Hermite polynomials, obtain the relation
 $H'_n(x) = 2nH_{n-1}$
- Show that $\Gamma(n+1) = n\Gamma n$, where n is an integer. (2 x 5 = 10)

Part C (Problem/Short essay)

(Answer any 3 question. Each question carries 4 Marks)

- Describe the Schimidt orthogonalization method for constructing an orthogonal set of functions from a non-orthogonal set.

15. Solve the following set of equations using Gauss elimination method

$$3x + 4y + 5z = 18$$

$$2x - y + 8z = 13$$

$$5x - 2y + 7z = 20$$

16. Write notes on Binomial and Poisson's distributions

17. If n is a positive integer and $J_{n(x)}$ is the Bessel function of n^{th} order, prove that

$$J_{n(x)} = \frac{1}{\pi} \int_0^{\pi} \cos(n\theta - x \sin \theta) d\theta$$

18. Show that $\epsilon_{ijk} \in \rho q k \epsilon_{ip} \delta_{jq} - \delta_{iq} \delta_{jp}$, where ϵ_{ijk} is the Levi-civita symbol (3 x 4 = 12)

Part D (Essay)

(Answer all question. Each question carries 12 Marks)

19. (a) State and prove Stokes theorem in vector analysis. Verify Stoke's theorem for the vector field $A = (3x-2y) \mathbf{i} + x^2z \mathbf{j} + y^2(z+1) \mathbf{k}$ for a plane rectangular area with vertices (0,0), (1,0), (1,2), (0,2) in the x-y plane

OR

b) Obtain the general expressions for vector operators in curvilinear coordinates and hence find the Laplacian in circular cylindrical coordinates.

20. (a) Explain matrix diagonalisation. Diagonalise the following matrix $\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

OR

(b) Find the characteristic equation of the matrices

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \text{ and verify Cayley Hamilton equation for them.}$$

Also find the inverse of each.

21. (a) What are Christoffel symbols? Show that they do not form the components of a tensor

Or

(b) Obtain the differential equation of a geodesic in a given space.

22. (a) Obtain the orthogonality relation for the Hermite polynomials

$$\int_{-\infty}^{+\infty} e^{-x^2} H_m(x) H_n(x) dx = \sqrt{\pi} 2^n n!$$

Or

(b) Obtain the series solutions of Legendre differential equation by Frobenius method

(12 x 4 = 48)
