P442

Reg.No...Name:..

MSc DEGREE END SEMESTER EXAMINATION MARCH 2016

SEMESTER – 4: MATHEMATICS

COURSE: **P4MATT20EL: CODING THEORY**

Time: Three Hours Max. Marks: 75

PART A

(Answer **any five** of the following. Each question carries 2 marks)

1. What is the minimum weight of the Hamming [7, 4] code? Is this Hamming [7, 4] code self orthogonal.
2. Find the parity check matrix for the (4, 2) ternary code with generator matrix $\begin{bmatrix} 1011 \\ 0121 \end{bmatrix}$
3. Prove that (23, 12, 7) binary code is perfect
4. If C is a binary (n, $\frac{n-1}{2}$) self orthogonal code, for odd n, then prove that c$^{\perp}$ is (n, $\frac{n+1}{2}$) code generated by $c \wedge h$.
5. If f(x) is a poly nominal with Coefficients in GF (P^{r}) them prove that $f\left(x^{pr}\right)=¿$
6. Show that x^2-2 is irreducible over GF (5).
7. Which binary cyclic codes of length 7 are self orthogonal?
8. Define a Vander Monde determinant and find its value.

(2 X 5 = 10)

PART B

(Answer **any five** of the following. Each question carries 5 marks)

9. Prove that the dual of an MDS Code C and is again an MDS Code.
10. If d is even prove that A (n-1, d-1) = A (n, d).
11. Describe the binary (24, 12) Golay Code. Show that it is self orthogonal, doubly even and triple error correcting.
12. Using double error correcting BCH Code decode the received vector y = (1,1,1,0, 1,1,1,1,1,1,1,1,1,1,1).
13. In a field F of characteristic Þ show that $(x \pm y)\, p^m = x^{p^m} \pm y^{p^m}$
14. Let m(x) be the minimal polynomial of an element α in a finite field GF (P^{r}) then prove the following
    1) m(x) is irreducible over GF (P)
    2) If α is a root of f(x) with coefficients in GF (P) then $m(x)/f(x)$
    3) $m(x)/x^{p^r}-x$

4) If m(x) is primitive then its degree is r. In every case degree of m (x) ≤ r.

15. If the degree of g (x) is n-k. Prove that C = < g(x) > has dimension k.

16. Define a Reed Solomon Code of designed distance d. Prove that it has d as its actual minimum weight. (5 x 5 = 25)

PART C

Answer either **Part A or Part B**. Each question carries 10 marks

17. a) Give a parity check matrix for the Hamming [7, 4] Code. Using Hamming decoding decode the messages (1,1,1,0,0,1,0) and (0,0,0, 1,1,1,0). Define the weight distribution of a Code. Find the weight distribution of the hamming (7, 4) Code.

b) Prove that the packing radius t has the following properties.

i. If C has minimum weight d, t = ¿]

ii. t is the largest among the numbers S so that each vector of weight ≤S is a unique coset leader.

18 a) Prove that the poly nomial $x^4 + x^3 + 1$ is irreducible over GF (2). Construct a field of order 16 using this polynomial and perform the following operations in this field.

i. 1001.1011 + 0101 ÷ 1100

ii. $(1110)^{1/2}$ + 1101

b) Every finite field has p^m elements for some prime P.

19. a) Prove that every cyclic (n, k) Code C has an idempotent generator e(x). Further the matrix M formed by e (x) and its next (k-1) cyclic shifts is a generator matrix for C.

b) Let C_1 and C_2 be cyclic codes with generator polynomials $g_1(x) \wedge g_2(x)$ and idempotent generators $e_1(x)$, and $e_2(x)$. Then $C_1 \cap C_2$ has as generator polynomial $l.c.m.(g_1(x), g_2(x))$ and as idempotent generator $e_1(x)e_2(x)$ and $C_1 + C_2$ has as generator polynomial $g.c.d.(g_1(x), g_2(x))$and as idempotent generator $e_1(x)+e_2(x)-e_1(x)e_2(x)$.

20. a) 1. Prove that the minimum weight of a BCH Code C of designed distance δ is atleast δ.

2. If C is a cyclic code over F whose generator polynomial g (x) has roots $\beta, \beta^2, \ldots \beta^{\delta-1}$ in some extension field of F then prove that minimum weight C is ≥ δ.

b) Let f(x) be a polynomial with coefficients in GF (q) and let S be the set of its roots in some field $F = GF(q^m)$ Then prove that weight of $f(x) \geq$ the size of any set A in a set 1, of students of F that is independent with respect to S.

(4 x 10 = 40)

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