Name:……………………………………… Reg. No…………………………………..

MSc DEGREE END SEMESTER EXAMINATION MARCH 2016

SEMESTER – 4: MATHEMATICS

COURSE: P4MATT19EL: THEORY OF WAVELETS

Time: Three Hours Max. Marks: 75

PART A

Answer **any five** questions. Each carries 2 marks

1. What is the necessary and sufficient condition for $\{R_k w\}_{k=0}^{N-1}$ is an orthonormal basis for $l^2(z_N)$ where $w \in l^2(z_N)$.
2. Distinguish between down sampling and upsampling operator with examples.
3. Explain " Wavelets" on Z_N
4. Suppose N is divisible by 2,and $u_1 \in l^2(Z_N)$.Define $u_2 \in l^2(Z_{N/2})$ b $u_2(n) = u_1(n) + u_1(n + \frac{N}{2})$, then prove that for all m,thefourier transforms $u_2(m) = u_1(2m)$.
5. Comment the statement ; $l^2(Z)$ is a Hilbert space.
6. Prove that the trigonometric system is an orthonormal set in $L^2([-\pi,\pi))$.
7. Define Pth stage wavelet syatemfor $l^2(Z)$.
8. Prepare a short note about translation bounded linear operator on $L^2([-\pi,\pi))$ with an example.

(2 x 5 = 10)

PART B

Answer **any five** questions. Each carries 5 marks

9. Prove that the Shannon wavelet basis is first stage wavelet basis for $l^2(Z_N)$.
10. Suppose $u = \{\sqrt{2}, \sqrt{2}, 0, 0\}$ and $v = \{0, 0, \sqrt{2}, \sqrt{2}\}$ .Prove that $\{v, R_2 v, u, R_2 u\}$ is an orthonormal basis for $l^2(Z_4)$.
11. If N is divisible by 2^l and $x, y, w \in l^2(Z^l_{N/2})$ and $z \in l^2(Z_N)$,then prove that $D^l(z) * w = D^l(z * w)$ where $D^l(z)$ means the composition of D with itself l times applied to z.
12. Give an algorithm for constructing a P[th] stage wavelet basis for $l^2(Z_N)$.

13. Suppose $f\in L^1([-\pi,\pi))$ and $\langle f,e^{in\theta}\rangle=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(\theta)e^{-in\theta}d\theta=0$ for all $n\in Z$, then prove that

$f(\theta)=0$ ***a.e***

14. Prove that $L^2([-\pi,\pi))<L^1([-\pi,\pi))$ with the inclusion can be strict

15. Define Complete ortthonormal system in a Hilbert space H.Also explain Parseval's relation and Plancherel's formula of a complete orthonormal system in a Hilbert space H.

16. What you meant by system matrix of u,v where $u,v\in l^2(Z)$.Discuss the importance of system matrix in a complete orthonormal set in $l^2(Z)$.

(5 x 5 = 25)

PART C

Answer **all** questions each carries 10 marks

17. A. (i) Define Conjugate reflection of $w\in l^2(Z_N)$.Also derive any one of its property.

(ii) Let $z,w,u,v\in l^2(Z_N)$,then prove that $\langle R_k z,R_j w\rangle=\langle z,R_{j-k}w\rangle=\langle R_{k-j}z,w\rangle$ for any $k,j\in Z$

OR

B. Suppose M∈N, N= 2M and $\omega\in l^2(Z_N)$Then $\{R_{2k}\}_{K=0}^{M-1}$ is an orthonormal set if and only if

$$|w(n)|^2+|w(n+M)|^2=2 \quad \text{for } n=0,1,2,......,M-1$$

18. A (i) Suppose $z,w\in l^2(Z_N)$. Prove that $(z*w)=\tilde{z}*\tilde{w}$

(ii) Suppose $z\in l^2(Z_{N/2})$ where N is even. Prove that $(u(z))=u¿$with standard notation.

OR

B. (i) If N is divisible by 2^p,define a Pth stage filter sequence .Find the output of the analysis

phase of the pth stage wavelet filter bank.

(ii) Explain orthogonol direct sum of inner product space X.

19. A. Let H be Hilbert space and $\{a_j\}_{j\in Z}$ be an orthonormal set in H. State and prove a necessary and sufficient condition for this set to be complete in H.

OR

B. Suppose $T: L^2([-\pi,\pi)) \to L^2([-\pi,\pi))$ is a bounded translalation – invariant linear transformation. Then prove that each $m \in Z$, there exists $\lambda_m \in C$ such that

$$T\left(e^{im\theta}\right) = \lambda_m e^{im\theta}$$

20. A. Define the convolution of z and w where $z, w \in l^2(Z)$ .Suppose that $z \in l^2(Z)$ and $w \in l^1(Z)$ then prove that $z * w \in l^2(Z)$ and $\|z * w\| \leq \|w\|_1 \|z\|$.

OR

B. Develop Harr wavelets on Z

(10 x 4 = 40)

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