Name:……………………………………Reg. No……………………………………

MSc DEGREE END SEMESTER EXAMINATION MARCH 2016

SEMESTER – 4: MATHEMATICS

COURSE: P4MATT19EL: THEORY OF WAVELETS

Time: Three Hours Max. Marks: 75

PART A

(Answer **any five** questions. Each carries 2 marks)

1. What is the necessary and sufficient condition for $\{R_k w\}_{k=0}^{N-1}$ is an orthonormal basis for $l^2(z_N)$ where $w \in l^2(z_N)$.
2. Distinguish between down sampling and upsampling operator with examples.
3. Explain " Wavelets" on Z_N
4. Suppose N is divisible by 2,and $u_1 \in l^2(Z_N)$.Define $u_2 \in l^2(Z_{N/2})$ b $u_2(n) = u_1(n) + u_1(n + \frac{N}{2})$,

   then prove that for all m,thefourier transforms $u_2(m) = u_1(2m)$.
5. Comment the statement ; $l^2(Z)$ is a Hilbert space.
6. Prove that the trigonometric system is an orthonormal set in $L^2([-\pi, \pi))$.
7. Define Pth stage wavelet syatemfor $l^2(Z)$.
8. Prepare a short note about translation bounded linear operator on $L^2([-\pi, \pi))$ with an example.

(2 x 5 = 10)

PART B

(Answer **any five** questions. Each carries 5 marks)

9. Prove that the Shannon wavelet basis is first stage wavelet basis for $l^2(Z_N)$.
10. Suppose $u = \{\sqrt{2}, \sqrt{2}, 0, 0\}$ and $v = \{0, 0, \sqrt{2}, \sqrt{2}\}$ .Prove that $\{v, R_2 v, u, R_2 u\}$ is an orthonormal basis for $l^2(Z_4)$.
11. If N is divisible by 2^l and $x, y, w \in l^2(Z^l{}_{N/2})$ and $z \in l^2(Z_N)$,then prove that $D^l(z) * w = D^l(z * w)$ where $D^l(z)$ means the composition of D with itself l times applied to z.
12. Give an algorithm for constructing a Pth stage wavelet basis for $l^2(Z_N)$.

13. Suppose $f \in L^1([-\pi,\pi))$ and $\langle f, e^{in\theta}\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)e^{-in\theta}d\theta = 0$ for all $n \in Z$,then prove that

$f(\theta) = 0$ ***a.e***

14. Prove that $L^2([-\pi,\pi)) < L^1([-\pi,\pi))$ with the inclusion can be strict

15. Define Complete ortthonormal system in a Hilbert space H.Also explain Parseval's relation and Plancherel's formula of a complete orthonormal system in a Hilbert space H.

16. What you meant by system matrix of u, v where $u, v \in l^2(Z)$.Discuss the importance of system matrix in a complete orthonormal set in $l^2(Z)$.

(5 x 5 = 25)

PART C

(Answer **all** questions each carries 10 marks)

17. A. (i) Define Conjugate reflection of $w \in l^2(Z_N)$.Also derive any one of its property.

(ii) Let $z, w, u, v \in l^2(Z_N)$,then prove that $\langle R_k z, R_j w\rangle = \langle z, R_{j-k} w\rangle = \langle R_{k-j} z, w\rangle$ for any $k, j \in Z$

OR

B. Suppose M∈N, N= 2M and $\omega \in l^2(Z_N)$Then $\{R_{2k}\}_{K=0}^{M-1}$ is an orthonormal set if and only if

$$|w(n)|^2 + |w(n+M)|^2 = 2 \quad \text{for } n = 0,1,2,......,M-1$$

18. A (i) Suppose $z, w \in l^2(Z_N)$. Prove that $(z*w) = \tilde{z}*\tilde{w}$

(ii) Suppose $z \in l^2(Z_{N/2})$ where N is even. Prove that $(u(z)) = u¿$with standard notation.

OR

B. (i) If N is divisible by 2^p ,define a P[th] stage filter sequence .Find the output of the analysis phase of the pth stage wavelet filter bank.

(ii) Explain orthogonol direct sum of inner product space X .

19. A. Let H be Hilbert space and $\{a_j\}_{j\in Z}$ be an orthonormal set in H. State and prove a necessary and sufficient condition for this set to be complete in H.

OR

B. Suppose $T : L^2([-\pi,\pi)) \to L^2([-\pi,\pi))$ is a bounded translalation – invariant linear transformation. Then prove that each $m \in Z$, there exists $\lambda_m \in C$ such that

$$T\left(e^{im\theta}\right)=\lambda_m e^{im\theta}$$

20. A. Define the convolution of z and w where $z, w \in l^2(Z)$ .Suppose that $z \in l^2(Z)$ and

$w \in l^1(Z)$ then prove that $z * w \in l^2(Z)$ and $\|z * w\| \leq \|w\|_1 \|z\|$.

OR

B. Develop Harr wavelets on Z

(4 x 10 = 40)

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