Reg.No...**Name:**.......................................

MSc DEGREE END SEMESTER EXAMINATION MARCH 2016

SEMESTER – 4, MATHEMATICS

COURSE: P4MATT17EL: COMBINATORICS

Time: Three Hours Max. Marks: 75

PART A

(Answer **any five** questions. Each carries 2 marks)

1. Find the number of distinct k-ary sequences of length n.
2. For n,r ϵ *N* and *r* ≤ *n*, show that $\binom{n}{r}=\binom{n-1}{r-1}+\binom{n-1}{r}$.
3. State and prove Pigeonhole principle.
4. Prove that at a gathering of any six people some 3 of them are mutual acquaintances or
completely strangers.
5. Show that $E(0)=\omega(0)-\omega(1)+\omega(2)+......+(-1)^q\omega(q)$.
6. Show that $D(n)=nD_{n-1}+(-1)^n$.
7. Let $S=\{s_1, s_2, ...s_n\}$. Let a_r be the number of ways of selecting r elements from S. Find a_r and the generating function of (a_r).
8. Show that the exponential generating function for the sequence (1, 1.3, 1.3.5, ...) is $(1-2x)^{-3/2}$.

(2 x 5 = 10)

PART B

(Answer **any five** questions. Each carries 5 marks)

9. Find the number of positive divisors of 600 including 1 and 600 itself.
10. Let S be the set of natural **numbers** whose digits are chosen from {1, 3, 5,7} such that no digits are repeated. Find
(i). |S|. (ii). $\sum_{n\in N} n$.
11. Let A= {a_1, a_2, a_3, a_4, a_5} be a set of 5 natural numbers. Show that for any permutation $a_{i1}a_{i2}a_{i3}a_{i4}a_{i5}$ of
A, the product ($a_{i1}-a_1$) ($a_{i2}-a_2$) ($a_{i3}-a_3$) ($a_{i4}-a_4$) ($a_{i5}-a_5$) is even.
12. Prove that *R(2,q)=q* for all *q* ϵ *N*.
13. Find the non-negative integer solutions to the linear equation $x_1+x_2+x_3=15$, where $x_1 \le 5$, $x_2 \le 6$, $x_3 \le 7$.
14. Suppose the numbers 1, 2, 3,......, m (m ≥ 3) are placed in order around a circle. For 0 ≤ k ≤ $\left[\frac{m}{2}\right]$. Let α(k) denotes the number of k element subsets of N_m in which no two elements are adjacent around the circle. Show that α(k) $=\frac{m}{k}\binom{m-k-1}{k-1}$.

15. Let S be the multi set S = {2.a, 1.b} and a_r denotes the number of ways of selecting r objects from S. Find a_r and the generating function for (a_r).
16. For each r ϵ *N**, find the number of ways of distributing r distinct objects into n distinct boxes such that no box is empty.

(5 x 5 = 25)

PART C

(Answer **all** questions each carries 10 marks)

17. a) Find the number of binary sequences of length 7 containing exactly 3 zero's and 4 one's.

    b) In how many ways can 5 boys and 3 girls be seated around a table if,

    (i). There is no restriction.

    (ii). Boy B_1 and girl G_1 are not adjacent.

    (iii). No girls are adjacent.

OR

18. a). Discuss the distribution problem of r identical objects into n indistinguishable boxes in the following 3 cases.

    i. Each box can hold at most one object.
    ii. Each box can hold any number of objects.
    iii. Each box holds at least one object.

    b). How many ways are there to arrange the letters of the word **VISITING** if no two **I**'s are adjacent.

19. a). Among any group of 3000 people, prove that there exist at least 9 having the same birthday.

    b). Let X <u>**C**</u> {1, 2, 3, ……,99} and |X|=10. Show that it is possible to select two disjoint non-empty proper subsets Y and Z of X such that $\sum y / y \in ¿ Y=\sum z / z \in Z ¿$.

OR

20. a). State generalized Pigeonhole Principle.

    b). Prove that , For all integers p, q ≥ 2, $R(p,q) \leq R(p-1,q)+R(p,q-1)$.

21. Let $A_1, A_2, \ldots\ldots A_q$ be any q subsets of the finite set S. Show that

$$|\overline{A_1} \cap \overline{A_2} \cap \ldots\ldots \overline{A_q}|=|S|-\sum_{i=1}^{q}|A_i|+\sum_{i<j}|A_i \cap A_j|-\sum_{i<j<k}|A_i \cap A_j \cap A_k|+\ldots+(-1)^q|A_1 \cap A_2 \ldots \cap A_q|$$

OR

22. There are n married couples n ≥ 3 to be seated in the 2n chairs around a table. Suppose that n wives have already been seated such that there is one and only one empty chair between two adjacent wives. Let M(n,r) denotes the number of ways to seat the n husbands in the remaining chairs such that exactly r husbands are adjacent to their wives. Show that

$$M(n,r)=\sum_{k=r}^{n}(-1)^{k-r}\binom{k}{r}\frac{2n}{2n-k}\binom{2n-k}{k}(n-k)!$$.

23. a). Show that the number of partitions of r into distinct parts is equal to the number of partitions of r into odd parts.

b). Solve $a_n - 7a_{n-1} + 15a_{n-2} - 9a_{n-3} = 0$, given $a_0 = 1, a_1 = 2$ *and* $a_3 = 3$.

OR

24. State and solve the problem of Tower of Hanoi.

(10 x 4 = 40)

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