**P403**

**Reg. No**.......................................**Name:**.......................................

# MSc DEGREE END SEMESTER EXAMINATION MARCH 2016

## SEMESTER – 4 MATHEMATICS

### COURSE: P4MATT16: SPECTRAL THEORY

Time: Three Hours Max. Marks: 75

### PART-A

(Answer **any five** of the following. Each question carries 2 marks)

1. Prove that strong convergence implies weak convergence with the same limit
2. Prove that a contraction T on a metric space is a continuous mapping
3. Find the eigenvalues of the matrix $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ where a, b are real with b≠ 0
4. Show that the set of all linear operators on a vector space into itself forms an algebra
5. Prove that every compact linear operator T: X→ Y is continuous
6. Define a symmetric linear operator
7. Let T: H→ H be a bounded self adjoint linear operator on a compact Hilbert space H. Prove that all the eigen values of T are real.
8. Define a positive operator. If $T_1 \leq T_2$ then prove that $\alpha T_1 \leq \alpha T_2$, where $\alpha$ is a scalar.

(2 x 5 = 10)

### PART B

(Answer **any five** of the following. Each question carries 5 marks)

9. Prove that the strongly operator convergent always implies weakly operator convergent but the converse is not generally true
10. Prove that closedness does not implies boundedness of a linear operator
11. Prove that two matrices representing the same linear operator T on a finite dimensional normed space relative to any two bases for X are similar
12. Prove that if X ≠{0} is a complex Banach space and T ε B(X, X), then $\sigma$(T) ≠$\varphi$.
13. Let T: X→X be a compact linear operator and S: X→X be a bounded linear operator on a normed space X then the operators TS and ST are compact.
14. State and prove Hellinger-Toeplitz Theorem
15. Prove that the spectrum $\sigma$(T) of a bounded self adjoint linear operator T:H→H on a complex Hilbert space H is real

16. Let $P_1 \wedge P_2$ be projections on a Hilbert space H. Then prove that P = $P_1 + P_2$ is a projection on H if and only if $Y_1 = P_1$(H) and $Y_2 = P_2$(H) are orthogonal.

(5 x 5 = 25)

## PART C

(All questions below are compulsory. Answer **(A)** or **(B)**. Each question carries 10 marks)

17.(A)(a) Let ($x_n$) be a weakly convergent sequence in a normed space X, say $x_n \xrightarrow{w}$ x .

Then the sequence ( $\|x_n\|$ ) is bounded

(b) Let ($T_n$) ε B(X,Y) where X is a Banach space and Y is a normed space. If ($T_n$) is strongly operator convergent with limit T, then prove that T ε B(X,Y)

(B) State and prove Closed Graph Theorem

18.(A) State and prove Spectral Mapping theorem for polynomials

(B)(a) Prove that the resolvent $R_\lambda$(T) of a bounded linear operator T: X → X on a

complex Bananch space is locally holomorphic on $\rho$(T).

(b) Let A be a complex Banach Algebra with identity e. If x ε A satisfies $\|x\|$< 1,

then prove that e-x is invertible and $(e-x)^{-1}$ = e + $\sum_{j=1}^{\infty} x^j$

19.(A) Let T:X→ Y be a linear operator, where X and Y are normed spaces. If T is a compact linear operator then prove that its adjoint operator $T^{\times}$: $Y' \rightarrow X'$ is also a compact linear operator

(B) (a) Let B be a subset of a metric space X. If B is relatively compact then prove that B is totally bounded

(b) Let T:X→ X be a compact linear operator on a Banach space X. Then prove that

every spectral value λ≠0 of T is an eigenvalue of T

20.(A) If two bounded self adjoint linear operators S and T on a Hilbert space H are

positive and commute(ST=TS) then their product ST is positive

(B) Let ($P_n$) be a monotonic increasing sequence of projections $P_n$ defined on a Hilbert

space H. Then prove that

(a) P projects H onto P(H) = ¿n=1¿∞ $P_n(H)$¿

(b) P has the null space N(P) = ¿n=1¿∞ $N(P_n)$

(10 x 4 = 40)

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