

B.Sc. DEGREE END SEMESTER EXAMINATION OCTOBER 2016
SEMESTER - 1: MATHEMATICS FOR BSC MATHEMATICS, COMPUTER
APPLICATIONS AND BCA

COURSE - 15U1CRMAT1-15U1CRCMT1-16U1CPCMT1: FOUNDATION OF
MATHEMATICS

Common for Regular (2016 Admission) & Supplementary / Improvement (2015 Admission)

Time: Three Hours

Max Marks: 75

Part AShort Answer Questions. Answer **all** questions.

Each Question carries 1 mark

- Define "ceiling of x " where X is a real number.
- Given $f : \mathbb{Z} \rightarrow \mathbb{N}$ is defined by
$$f(x) = \begin{cases} 2x-1, & \text{if } x > 0 \\ -2x, & \text{if } x \leq 0 \end{cases}$$

Find $f^{-1}(x)$

- Prove or disprove that $[x + y] = [x] + [y]$ for all real numbers x and y .
- Evaluate $\sum_{i=1}^3 \square \sum_{j=1}^2 ij$
- Define factorial function.
- Negate the statement $\forall x \exists y [p(x) \vee q(y)]$
- Define "Inverse of the conditional statement": $p \rightarrow q$.
- State Fundamental theorem of Arithmetic.
- Prove that 284 and 220 are amicable numbers.
- If $n = P_1^{m_1} P_2^{m_2} P_3^{m_3} \dots$ then obtain ϕ
Where P_1, P_2, P_3 are distinct primes.

(1 x 10 = 10)

Part BBrief Answer Questions. Answer **any eight** questions.

Each Question carries 2 marks

- Suppose $A = \{x, y, z\}$. Obtain the power set of A .
- Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 2x$.
Find **(fof)2** and **(fof)3**.
- Given $f : \mathbb{Z} \rightarrow \mathbb{N}$ is defined by $f(x) = \begin{cases} 2x-1, & \text{if } x > 0 \\ -2x, & \text{if } x \leq 0 \end{cases}$

Prove that f is one- to -one

14. Define "Partition of a set S".
Write down two possible partitions of the set $S = \{a,b,c,d,e,f,g,h\}$.
15. Show that $(p \rightarrow q) \wedge (q \rightarrow p)$ and $(p \leftrightarrow q)$ are logically equivalent.
16. Find the smallest number with 10 divisors.
17. If $n=ab$ where $(a,b)=1$, Show that $\phi(a,b) = \phi(a) \phi(b)$.

18. If N be any integer, n the number of its divisors and P the product of them all. Prove that $N^n = P^2$
19. Find the g.c.d of 58 and 86 and express it as a linear combination of the above two integers
20. Negate the following Statements
(1) $(\forall x \in A)(x + 2 = 7)$
 $(\exists x \in A)(x + 2 \geq 7)$ (2 x 8 =16)

Part C

Short Essay type Questions. Answer **any five** questions.
Each Question carries 5 marks

21. Compute the value of $\sum_{k=50}^{100} k^2$
22. Let A be a set of non zero integers and let $*$ be the relation on $A \times A$ defined by $(a,b) * (c,d)$ whenever $ad = bc$. Prove that $*$ is an equivalence relation.
23. Find the join and meet of the zero-one matrix

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{matrix} 1 \\ 0 \end{matrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{matrix} 0 \\ 0 \end{matrix}$$

24. Show that $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$.
25. If $p \rightarrow q$ and $q \rightarrow r$ then show that $(p \rightarrow r)$ is a tautology.
26. Show that $3^{2n+1} + 2^{n+2} = M(7)$
27. Prove that cube of any number is of the form $7m$ or $7m \pm 1$
(5 x 5 =25)

Part D

(Essay). Answer **any two** questions. Each Question carries 12 marks

28. Prove that that $(A - C) \cap (C - B) = \phi$ analytically where A, B & C are sets. Also verify graphically.
29. Let R_5 be the relation on the set Z of integers defined by $x R_5 y$ if $x \equiv y \pmod{5}$. Show that R_5 is an equivalence relation

Give a proof by contradiction of the theorem “ if n^2 is even then, n is even “.

30. (a) State and prove Wilson’s theorem.

(b) Show that

$$18! + 1 \equiv 0 \pmod{23}$$

(12 x 2 =24)
