Reg. No……..…..…...….……...……… Name…………..…..……… **U 15123**

BSc DEGREE EXAMINATION - OCTOBER 2015

SEMESTER - 1: MATHEMATICS CORE COURSE FOR BSC MATHEMATICS /

BSC COMPUTER APPLICATION .

COURSE - 15U1CRMAT1–15U1CRCMT1: FOUNDATION OF MATHEMATICS

Time: Three Hours Max. Marks: 75

Part A

Short Answer Questions. Answer all questions. Each Question carries 1 mark

1. Define " floor of *x* " *where x is a real number.*
2. *Given* $f : Z \to N$ is defined by $f(x) = \begin{cases} 2x-1, & \text{if } x > 0 \\ -2x, & \text{if } x \le 0 \end{cases}$

   Find $f^{-1}(x)$
3. Prove or disprove that [x + y] = [x] + [y] for all real numbers x and y.
4. Evaluate $\sum_{i=1}^{4} \square \sum_{j=1}^{3} ij$
5. Define factorial function.
6. Negate the statement $\exists x \forall y\, [p(x,y) \to q(x,y)]$
7. Define , converse of the conditional statement : $p \to q$.
8. State Unique Factorization theorem
9. Define Amicable numbers
10. If $a \equiv b \pmod n$ and $a_1 \equiv b_1 \pmod n$. Show that $a a_1 \equiv b b_1 \pmod n$.

(1 x 10 = 10)

Part B

Brief Answer Questions. Answer any ***eight*** questions. Each Question carries 2 marks

11. Suppose S ={ 1, 2, 3 }. Write down the power set of S.
12. Let $f : R \to R$ be defined by $f(x) = x^2 + 2x$

    Find (fof)2 and (fof)3.
13. *Given* $f : Z \to N$ is defined by $f(x) = \begin{cases} 2x-1, & \text{if } x > 0 \\ -2x, & \text{if } x \le 0 \end{cases}$

    Prove that **f** is one- to –one
14. Define "Partition of a set S ".

Write down two possible partitions of the set S = {1,2,3,4,5,6,7,8}.

15. Show that $p \rightarrow q$ and $(\sim p \vee q)$ are logically equivalent.

16. Find the smallest number with 18 divisors.

17. If n=ab where (a,b) =1, Show that φ(a,b) = φ(a) φ(b).

18. Define perfect number. Give an example.

19. Show that **a** and **b** are relatively prime iff **1** is expressible as a liner combination of **a** and **b**.

20. Negate the following Statements

(1) $(\exists x \varepsilon A)(x + 3 < 10)$

(2) $(\forall x \varepsilon A)(x + 3 \geq 7)$

(8 x 2 =16)

Part C

(Short Essay type Questions). Answer any ***five*** questions. Each Question carries 5 marks

21. Compute the following

(a) $\sum_{j=0}^{5} j^2$

(b) $\sum_{k=4}^{8} (-1)^k$

22. Let **A** be a set of non zero integers and let * be the relation on A x A defined by (a,b) * (c,d) whenever ad = bc. Prove that * is an equivalence relation.

23. Find the join and meet of the zero-one matrix

$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

24. If $p \rightarrow q$ and $q \rightarrow r$ then show that $(p \rightarrow r)$ is a tautology.

25. Show that $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$.

26. Show that 7^{2n+1} **+1 = M (8)**

27. Show that every square is of the form $3m$ or $3m + 1$

(5 x 5 =25)

Part D

(Essay). Answer any ***two*** *questions. Each Question carries 12 marks*

28. If A, B, C and D are sets prove algebraically that

$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

Give an example to support this result.

29. Let R_5 be the relation on the set Z of integers defined by $x R_5 y$ if $X \equiv y \pmod 5$. Show that R_5 is an equivalence relation on Z.

30. Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction

31. (a) State and prove Fermat's Little theorem

(b) If **p** is an odd prime and **a** is prime to **p** show that $a^{(p-1)/2} \equiv \pm 1 \pmod p$

(12 x 2 =24)

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