BSc DEGREE EXAMINATION - OCTOBER 2015
SEMESTER- 1: MATHS COMPLEMENTARY FOR B.SC PHYSICS / B.SC CHEMISTRY
COURSE- 15U1CPMAT1: DIFFERENTIAL CALCULUS AND TRIGNOMETRY

Time: Three hours   Max. Marks: 75

Part A
Short Answer Questions. *Answer **all** questions.*
*Each question carries **one** mark.*

1. Find $\lim\limits_{x \to 1}\left(\dfrac{x^3-1}{x^2-1}\right)$.

2. State the Sandwich Theorem.

3. If $g(t)=\dfrac{1}{t^3}$, find $g'(\sqrt{3})$.

4. Find left hand limit of the function u(x) at x=0 ,

   where u(x)= $\begin{cases} 0, & x<0 \\ 1, & x \geq 0 \end{cases}$.

5. If $f'(x)=g'(x)$ at each point x in (a, b) then $f-g$ is _____

6. Write a chain rule formula to find $\dfrac{dz}{dt}$ for $z=f(x,y), x=g(t), y=h(t)$

7. Find $\dfrac{\partial f}{\partial x}$ if $(x,y)=\dfrac{2}{x^2+y^2}$.

8. The period of sin x is _____

9. Write the relation connecting the circular sine function and hyperbolic sine function.

10. State De Moivre's Theorem.

(1 × 10 = 10)

Part B
Brief Answer Questions. *Answer **any eight** questions.*
Each question carries two marks.

11. Find $\lim\limits_{x \to 0}\dfrac{\cos x-1}{x}$

12. Find the parametrization of the line segment with end points (-2,1) and (3,5)

13. If y = 2u^3 , u = 8x-1 , then find dy/dx.

14. If $x+\sin y=xy$, find $\dfrac{dy}{dx}$.

15. Find the function f(x) whose derivative is 2x-1 and passing through (0, 0).

16. If z is a function in x and y, find $\dfrac{\partial z}{\partial x}$, where yz-lnz = x + y .

17. Express $\dfrac{\partial w}{\partial r}$ in terms of r where $w=x^2+y^2, x=r-s, y=r+s$.

18. Show that $\sin 5\theta=5\sin\theta-20\sin^3\theta+16\sin^5\theta$.

19. Prove that cosh2x = cosh^{2}x+sinh^{2}x.

20. If x is real show that $\sinh^{-1}x = \log\left(x+\sqrt{x^2+1}\right)$.

$(2 \times 8 = 16)$

Part C

Short Essay Type Questions. *Answer any five questions.*
Each question carries 5 marks.

21. Find a δ > 0, to show that $\lim\limits_{x \to 5} \sqrt{x-1} = 2$ that works for ε = 1

22. Find the equation of the tangent and the normal to the curve $x^3 + y^3 - 9xy = 0$ at (2,4)

23. State and prove Mean Value Theorem.

24. Evaluate $\dfrac{dw}{dt}$ at $t=3$: where $w = \dfrac{x}{z} + \dfrac{y}{z}, x = \cos^2 t, y = \sin^2 t, z = \dfrac{1}{t}$.

25. Prove that if $y^3 - 3ax^2 + x^3 = 0$, then $\dfrac{d^2 y}{dx^2} + 2\dfrac{a^2 x^2}{y^5} = 0$

26. Separate $\tan^{-1}(x+iy)$ into real and imaginary parts.

27. If sin(A + iB) = x + iy, show that

$$\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1 \quad \text{and} \quad \frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1.$$

$(5 \times 5 = 25)$

Part D
Essay. *Answer **any two** questions.*
*Each question carries **12** marks.*

28. a) Differentiate $f(x) = \dfrac{x}{x-1}$ and find where does the curve have slope -1
    b) Prove that if f has a derivative at x = c, then f is continuous at x= c.
       Is the converse true? Give an example.

29. Find the critical points of f(x) = x^3-12x-5 . Identify the intervals on which f is increasing and decreasing. Find the local maximum and minimum values of the function.

30. a) Verify mixed derivative theorem for $f(x,y) = \left(\dfrac{xy}{x+y}\right)$.

b) If $z = e^x(x \cos y - y \sin y)$, prove that $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$

31. Sum the series

$$c\sin\theta + \frac{c^2 \sin 2\theta}{2} + \frac{c^3 \sin 3\theta}{3} + \dots \dots \text{ where } |c| < 1$$

(12 × 2 = 24)

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