Reg. No............................ Name............................ Qcode 14P2043

M. Sc. DEGREE END SEMESTER EXAMINATION APRIL 2017

SEMESTER **– 2: MATHEMATICS**

COURSE: **P2MATT09: PARTIAL DIFFERENTIAL EQUATIONS**

(Supplementary for 2014 admission)

Time: Three Hours Max. Marks: 75

PART A

Answer **any FIVE** questions; 2 marks each

1. Verify that the differential equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable.
2. Find the integral surface of the equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ which passes through the line $x_0(s) = 1,\ y_0(s) = 0$ *and* $z_0(s) = s$.
3. Along every characteristic strip of the equation $F(x, y, z, p, q) = 0$, the function $F(x, y, z, p, q)$ is a constant.
4. Find a complete integral of the equation $(p^2 + q^2)\, y = qz$.
5. Define hyperbolic, parabolic and elliptic equations. Give example of a parabolic equation.
6. Reduce the equation $u_{xx} + x^2 u_{yy} = 0$ to a canonical form.
7. Write the Monge's equations for the nonlinear equation $y^2 r - 2ys + t = p + 6y$.
8. Solve the one dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ by separating the variables.

(2 x 5 = 10)

PART B

Answer **any FIVE** questions; 5 marks each

9. Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1,\ z = 1$.
10. Verify that the differential equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and find its primitive.
11. Find a complete integral of the equation $p^2 x + q^2 y = z$ by Jacobi's method.
12. Show that a complete integral of $(u_x, u_y, u_z) = 0$ is $u = ax + by + cz + d$ where $f(a, b, c) = 0$. Hence find the complete integral of $u_x + u_y + u_z - u_x u_y u_z = 0$.
13. Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$
14. Reduce the equation $\frac{\partial^2 u}{\partial x^2} = x^2 \frac{\partial^2 u}{\partial y^2}$ to canonical form.
15. Prove that for the equation $\frac{\partial^2 u}{\partial x \partial y} + \frac{1}{4}u = 0$, the Riemann function is $v(x, y; \alpha, \beta) =$ ______________

$J_0(\sqrt{(x-\alpha)(y-\beta)}$ where J_0 denote the Bessel's function of the first kind of order zero.

16. Derive the condition that the surface $f(x, y, z) = c$ form a family of equipotential surfaces.

(5 x 5 = 25)

PART C

*Answer **either part (a) or part (b)**. Each carries 10 marks.*

17. (A) Prove that the Pfaffian differential equation $\vec{X}.d\vec{r}=0$ is integrable if and only if $\vec{X}$.Curl $\vec{X}= 0$.

OR

(B)Prove that if $u_i(x_1, x_2,\ldots,x_n, z) = c_i$ $(i = 1,2,3, \ldots \ldots, n)$ are independent solutions of the equations $\frac{dx_1}{P_1} = \frac{dx_2}{P_2} = \cdots . = \frac{dx_n}{P_n} = \frac{dz}{R}$, then the relation $\varnothing(u_1, u_2, \ldots \ldots, u_n) = 0$ in which the function $\varnothing$ is arbitrary, is a general solution of the linear partial differential equation $P_1\frac{\partial z}{\partial x_1}+ P_2\frac{\partial z}{\partial x_2} + P_3\frac{\partial z}{\partial x_3} + \cdots \ldots \ldots + P_n\frac{\partial z}{\partial x_n} = R$.

18. (A) Find the complete integral of the equation $p^2x + qy = z$ and derive the equation of the integral surface containing the line $y = 1$, $x + z = 0$ is a generator.

OR

(B) Describe Jacobi's method. Solve the equation $z^2 + zu_z - u^2_x - u^2_y = 0$ by Jacobi's method.

19. (A) Solve the equation

$$\frac{\partial^2 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2\partial y} - \frac{\partial^3 z}{\partial x\partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$$

OR

(B)Reduce the equation $y^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x\partial y} + x^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ to canonical form and hence solve it.

20. (A) Describe Monge's method. Solve $r = t$.

OR

(B) Define Riemann function. Prove that for the equation $\frac{\partial^2 u}{\partial x \partial y} + \frac{1}{4} u = 0$ the Riemann

function $(x, y ; \alpha, \beta) = J_0\sqrt{(x - \alpha)(y - \beta)}$ where $J_0(z)$ denote Bessel's function of the first kind of order zero. (10 x 4 = 40)

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