Reg. No.............................. Name............................ Qcode: 14P2018

M. Sc. DEGREE END SEMESTER EXAMINATION APRIL 2017

SEMESTER **– 2: MATHEMATICS**

COURSE: **P2MATT07: ADVANCED TOPOLOGY**

(Supplementary for 2014 admission)

Time: Three Hours Max. Marks: 75

PART A

*Answer **any five** questions. Each question carries **2** marks.*

1. State Urysohn's lemma.
2. Prove that projection functions are open.
3. Prove that T_1 axiom is a productive property.
4. Show that the evaluation function of a family of functions is one-to-one if and only if the family distinguishes points.
5. Prove that in an indiscrete space, every net converges to every point.
6. Let S be a family of subsets of a set X. Prove that there exists a filter on X having S as a sub-base if and only if S has the finite intersection property.
7. Show that a first countable, countably compact space is sequentially compact.
8. Show that every locally compact, Hausdorff space is regular.

(2 x 5 = 10)

PART B

*Answer **any five** questions. Each carries **5** marks.*

9. Suppose a topological space X has the property that for every closed subset A of X, every continuous real valued function on A has a continuous extension to X. Then prove that X is normal.
10. Prove that a product of topological spaces is completely regular if and only if each coordinate space is so.
11. Let $\mathbf{f_1, f_2, f_3 : R \to R}$ be defined by $\mathbf{f_1(x)=cosx, f_2(x)=sinx, f_3(x)=x}$ **for** $\mathbf{x \in R}$. Describe the evaluation maps of the families $\mathbf{\{ f_1, f_2\}}$, $\mathbf{\{ f_1, f_3\}}$ **and** $\mathbf{\{ f_1, f_2, f_3\}}$. Which of these families distinguish points?
12. Let $S:D\to X$ be a net in a topological space and let $x \in X$. Show that x is a cluster point of S if and only if there exists a subnet of S which converges to x in X.
13. Prove that every filter is contained in an ultra-filter.
14. Show that an ultra filter converges to a point if and only if that point is a cluster point of it.
15. Let X be a Hausdorff space and Y be a dense subset of X. Show that if Y is locally compact in the relative topology on it, then Y is open in X.
16. Show that if **Y** is a compact Hausdorff space and y_0 is any point of **Y**, then **Y** is homeomorphic to the Alexandroff compactification of the space $\mathbf{Y-\{ y_0 \}}$.

(5 x 5 = 25)

PART C

*Answer **either part (a) or part (b).** Each carries **10** marks*

17. (a) Show that a topological product is T_0, T_1, T_2 or regular if and only if each coordinate space has the corresponding property.

(b) Show that a product of spaces is connected if and only if each coordinate space is connected.

18. (a) State and prove Embedding Lemma.

(b) Prove that a 2^{nd} countable and T_3- space is embeddable in the Hilbert cube and hence it is metrisable.

19. (a) Let **X** and **Y** be topological spaces, $\mathbf{x_0} \in \mathbf{X}$ and $\mathbf{f:X \to Y}$ a function. Show that f is continuous at x_0 if and only if whenever a net S converges to x_0 in **X**, the net $\mathbf{f \circ S}$ converges to $\mathbf{f(x_0)}$ in **Y**.

(b) For a topological space **X**, show that the following statements are equivalent:

(i) **X** is compact

(ii) Every net in **X** has a cluster point in **X.**

(iii) Every net in **X** has a convergent subnet in **X**.

20. (a) Show that every countably compact metric space is second countable.

(b) Show that the one point compactification of a space is Hausdorff if and only if the space is locally compact and Hausdorff.

(10 x 4 = 40)

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