Reg. No............................ Name.................................. Qcode.14P2003

M. Sc. DEGREE END SEMESTER EXAMINATION APRIL 2017

SEMESTER **– 2: MATHEMATICS**

COURSE: **P2MATT06: ABSTRACT ALGEBRA**

(Supplementary for 2014 admission)

Time: Three Hours Max. Marks: 75

PART A

Answer **any five** questions. Each question carries **2** marks.

1. State the fundamental theorem of finitely generated abelian groups.
2. In $Z_n[x]$, the equation $x^2 - 1 = 0$ has exactly two solutions. True? Justify.
3. Check whether 3 is a primitive 6^{th} root modulo 7.
4. Define constructible numbers.
5. Are Q(√2) and Q(3+√2) two different extensions of the field Q?
6. Define Frobenius automorphism
7. Is the Galois field GF (2^8) perfect? Justify.
8. Define the Galios group.

(2 x 5 = 10)

PART B

Answer **any five** questions. Each question carries **5** marks.

9. Define the p^{th} Cyclotomic polynomial. Check whether it is irreducible over Q.
10. State and prove the Eisenstein Criterion.
11. Let F be a field and α be algebraic over F. Let the degree of irr (α, F) be $n \geq 1$. Prove that every element β of F(α) can be uniquely expressed as $\beta = b_0 + b_1 \alpha + \ldots + b_{n-1} \alpha^{n-1}$, $b_i \in F$.
12. Prove that every finite extension field of a finite field is a simple extension.
13. Prove that complex zeros of polynomials with real coefficients occur in conjugate pairs.
14. Let p be a prime number. Let G be a finite group and let p divide |G|. Prove that G has an element of order p.
15. If $f(x)$ is irreducible in F[x], prove that all zeros of $f(x)$ have same multiplicity in the closure of F.
16. State and prove the Primitive Element Theorem.

(5 x 5 = 25)

PART C

Answer **either Part I or Part II** of each question. Each question carries **10** marks.

17. (I) (a) Define the direct product groups. Introduce a group structure in the direct product.

(b) State and prove the necessary and sufficient condition for $Z_m \times Z_n$ to be a cyclic group.

(II) Let F be a subfield of a field E and let $\alpha \in E$. Define $\varnothing_\alpha : F[x] \to E$ by $\varnothing_\alpha (a_0 + a_1 x + \ldots + a_n x^n) =$

$a_0 + a_1 \alpha + \ldots + a_n \alpha^n$ Prove that $\varnothing_\alpha$ is a homomorphism. Find all the zeros in Z_5 of

$2x^{219} + 3x^{74} + 2x^{57} + 3x^{44}$.

18. (I) Prove that every field has an algebraic closure.

(II) Construct the Galois field $GF(2^3)$ containing 8 elements. Find the multiplicative inverses of any three nonzero elements of $GF(2^3)$.

19. (I) (a) Show that every group of order 255 is abelian and cyclic.

(b) Show that no group of order 48 is simple.

(II) (a) Let *E* be a finite extension of a field *F*. Show that the number extensions of an isomorphism of *F* onto a field F' to an iso-morphism of *E* onto a subfield of of $\acute{F}'$ is finite and is completely determined by the two fields *E* and *F*.1

(b) Express $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ as a simple extension.

20. (I) (a) Prove that every field of characteristic zero is perfect.

(b) Let E be a finite separable extension of a field F. Prove that E = F(α) for some α in E.

(II) (a) Prove that the Galois field $GF(p^n)$ is perfect.

(b) Let E be a splitting field over a field F. Prove that every irreducible polynomial in F[*x*] has a zero in E then all its zeros are in E. `

(10 x 4 = 40)

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