Reg. No............................ Name: .... **Q. Code: P150**

M COM DEGREE END SEMESTER EXAMINATION 2014 -15
SEMESTER -1: MATHEMATICS
COURSE: P1MATT05 - COMPLEX ANALYSIS – 1

Time: 3 Hours   Max. Marks: 75

Part A

Answer any **5** questions, each carries 2 marks.

1. Show that an analytic function defined in a region Ω reduces to a constant if its modulus is constant.
2. Find the fixed points and normal form of the transformation $w=\frac{z}{2-z}$
3. Show that $\left| \int_a^b f(t)\,dt \right| \leq \int_a^b ¿ f(t)|$ dt holds for arbitrary complex valued function $f:[a¿¿1b] \to C¿$
4. Evaluate $\int_\gamma x\,dz$ where γ is the directed line segment from 0 to 1+i
5. Show that zeros of an analytic function defined on a region are isolated.
6. Define meromorphic function. Give an example.
7. Show that any harmonic function which depends only on r must be of the form a log r +b.
8. Show that $P_{u+v} = P_u + P_v$

(5 x 2 = 10)

Part B

Answer any **5** questions, each carries 5 marks.

9. Define cross ratio. Prove that if z_1, z_2, z_3, z_4 are distinct points in the extended plane and T any linear transformation then$(TZ_1\, Tz_2 Tz_3 Tz_4)=(z_1,z_2,z_3,z_4)$.
10. Discuss the transformation $\omega=z^2$
11. State and prove cauchy’s integral formula.
12. Evaluate $\int_{|z|=\rho} |z-a|^{-4} \vee dz \vee ¿¿$, $|a| \neq \rho$
13. Show that an analytic function comes arbitrarily close to any complex value in the neighbourhood of an essential singularity.
14. Let z_j be the zeros of a function f(z) which is analytic in a disk Δ and does not vanish identically, each zero being counted as many times as its order indicates. For any closed curve γ in Δ which does not pass through zero $\sum_j n(\gamma, z¿¿ j)¿ = \frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}$dz where the sum has only a finite number of terms not equal to zero. Prove.

15. Prove that all the roots of $z^7-5z^3+12=0$ lie between the circles $|z|=1$ and $|z|=2$

16. State and prove cauchy's residue theorem. (5 x 5 = 25)

Part C

Answer **either (a) or (b)** of the following **four** questions.

17. a) prove that (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

OR

b) Find a linear transformation which carries the circle $|z|=2$ into $|z+1|=1$, the point -2 into the origin and the origin to i

18 a) State and prove cauchy's theorem for a rectangle

OR

b) (1) Let f(z) be analytic in the region Δ' obtained by omitting a finite number of points ζ_j from an open disk Δ. If f(z) satisfies the condition $\lim_{z \to \zeta_j} (z-\zeta_j) f(z)=0$ for all j then $\int_\gamma f(z)dz=0$ holds for any closed curve γ in Δ'. Prove.

(2) Show that if the piecewise differentiable closed curve γ does not pass through the point a then the value of the integral $\int_\gamma \frac{dz}{z-a}$ is a multiple of $2\pi i$

19. a) State and prove Taylor's theorem

OR

b) Prove that if p dx + q dy is locally exact in Ω then $\int_\gamma p\,dx+q\,dy=0$ for every cycle γ homologus to zero in Ω.

20. a) If u_1 and u_2 are harmonic in a region Ω then $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$ for every cycle γ which is homologus to zero in Ω. Prove.

OR

b) Using residue theorem prove that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$

(10 x 4 = 40)

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