Reg. No............................... Name: .... **Q. Code: P129**

M SC DEGREE END SEMESTER EXAMINATION 2014 -15
SEMESTER -1: MATHEMATICS
COURSE: P1MATT03 - MEASURE THEORY AND INTEGRATION

Time: 3 Hours Max. Marks: 75

Part A

Answer **any five** questions. Each question carries 2 marks

1. Show that the outer measure of a countable set is zero.

2. If m*(E)=0, then show that E is measurable.

3. Let f be a non negative measurable function. Show that $\int_E f = 0$ implies f=0 a.e. in E.

4. Show that if f(x) = 0, x irrational 1, x rational

Then $R\overline{\int_a^b} f(x)dx = b-a$ and $R\int_{\underline{a}}^{b} f(x)dx = 0$.

5. If f and g are bounded measurable functions defined on a set E of finite measure then show that

$$\int_E af+bg = a\int_E f + b\int_E g$$

6. If A ϵ B, B ϵ B, and A C B, then show that μ (A)≤ μ(B), where B is the σ – algebra generated by the set of subsets of X.

7. Define a positive set. Show that the countable union of positive sets is positive.

8. If a sequence of measurable functions converges in measure then show that the limit function is unique almost everywhere.

2 x 5 = 10

Part B

Answer any **five** questions. Each question carries 5 marks

9. Show that the outer measure is countably sub additive.
10. If *f* and *g* are two measurable functions, then show that *f* + *g*, *f.g* are measurable.
11. Show that if *f* is a measurable function and *f* = *g* a.e., then *g* is measurable.
12. Let < f_n> be a sequence of measurable functions defined on a set *E* of finite measure and suppose that there is a real number M such that $|f(x)| \leq$ M for all *n* and *x*. If $f(x) = \lim f_n(x)$ for each *x* in *E*, show that $\int_E f = \lim \int_E f_n$.
13. If *f* and *g* are bounded measurable functions defined on a set *E* of finite measure, show that

(a) If *f* = g a.e., $\int_E f = \int_E g$

(b) If *f* ≤ g a.e., $\int_E f \leq \int_E g$

14. State and prove Hahn decomposition theorem.

15. Show that if E ϵ S XT, then for each x ϵ X and y ϵ Y, E_x ϵ T and E^y ϵ S.

16. If A ϵ **a**, where **a** is a σ-algebra on some non empty set X, then show that A is measurable with respect to μ*.

5 x 5 = 25

Part C

Answer either part I or part II of all questions. Each question carries 10 marks

17. I.(a). Show that the outer measure of an interval equals to its length

(b). Show that the interval (a, ∞) is measurable.

II. (a). Show that every Borel set is measurable.

(b). Let $<E_n>$ be an infinite decreasing sequence of measurable sets, that is a sequence with $E_n \subset E_{n+1}$ for each n. Then **s**how that $m\left(\underset{i=1}{\overset{\infty}{intersect}} E_i\right)=\lim_{n\to\infty} m(E_n)$.

18.I. (a). State and prove Fatou's lemma.

(b). State and prove Monotone Convergence theorem.

II. Let f be an increasing real valued function on the interval [a,b]. Then show that f is differentiable almost everywhere, the derivative f' is measurable and $\int_a^b f'(x)\,dx \le f(b)-f(a)$.

19. I. (a). Suppose that for each α in a dense set D of real numbers there is assigned a set B_α ϵ B such that $\mu(B_\alpha - B_\beta)=0$ for α<β. Then show that there is a measurable function f such that $f \le \alpha$ a.e. on B_α and $f \ge \alpha$ a.e. on X- B_α. If g is any other function with this property, show that $g = f$ a.e.

(b). Let E be a measurable set such that 0< ν(E)<∞. Then show that there is a positive set A contained in E with ν(A)>0.

II. State and prove Caratheodory Theorem.

20. I.(a). If $\{f_n\}$ is a sequence of measurable functions which is fundamental in measure, then show that there exist a measurable function f such that $f_n \to f$ in measure.

(b). Let $f_n \to f$ in measure, where f and each f_n are measurable functions. Then show that there is a subsequence $\{n_i\}$ such that $f_{n_i} \to f$ a.e.

II. (a). If A is an algebra, then show that the algebra generated by A is the smallest monotone class containing A.

(b). Let f be a non negative S XT- measurable function and let

$$\varphi(x)=\int_Y f_x\,dv,\ \psi(y)=\int_X f^y\,d\mu$$

for each xϵX, yϵY; then show that φ is S- measurable, ψ is T- measurable and

$$\int_X \varphi\,d\mu=\int_{XXY} f\,d(\mu X v)=\int_Y \psi\,dv$$

4 x 10 = 40

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