Reg. No............................. Name: .... **Q. Code:** P104

M SC DEGREE END SEMESTER EXAMINATION 2014 -15
SEMESTER -1: PHYSICS
COURSE (CODE): P1PHYT01 TITLE: MATHEMATICAL METHODS IN PHYSICS-1

Time: 3 Hours. Max. Marks: 75

Part A (Objective Type)

*(Answer **all questions**. Each question carries 1 mark)*

1. A rigid body is rotating about a fixed axis with a constant angular velocity $\vec{\omega}$. If $\vec{v}=\vec{\omega}\times\vec{r}$; what is $\nabla\times\vec{v}$?
   (a) ω (b) 2ω (c) 0 (d) $r\omega$
2. The linear vector space of all complex numbers over a real field is
   (a) One dimensional (b) two dimensional (c) three dimensional (d) infinite dimensional
3. If $A=\begin{pmatrix} 1 & \sqrt{2} \\ -\sqrt{2} & -1 \end{pmatrix}$. Then A^3 is (a) A (b) $-A$ (c) I (d) $-I$
4. Which of the following is the value of $\beta(1,2)$?
   (a) 0 (b) 0.5 (c) 0.05 (d) 1
5. The number of independent components of an antisymmetric tensor of rank 2 in 3-dimensional space is (a) 2 (b) 3 (c) 4 (d) 6

(5 × 1 = 5)

Part B (short Answer)

*(Answer **any 5** questions. Each question carries 2 marks)*

6. Suppose V is a vector space of which S is subspace. Prove that all the vectors of V which are orthogonal to S form a linear vector space.
7. Find out the most general orthogonal square matrix of order 2.
8. Show that the process of contraction of a tensor produces another tensor but with an order reduced by 2.
9. Show that $\nabla f(r)=\frac{\vec{r}}{r}\frac{df}{dr}$ where $f(r)=f$¿
10. What are the properties of Hermitian Operators?
11. Show that $\Gamma\left(\frac{1}{2}-n\right)\Gamma\left(\frac{1}{2}+n\right)=(-1)^n\pi$
12. Define Dirac delta function and show that $\int_{-\infty}^{\infty} f(x)\delta(x-a)\,dx=f(a)$
13. When studying special functions, why do we give so much importance to their orthogonality properties?

(5×2=10)

Part C (Problem/short essay)

*(Answer **any 3** questions. Each question carries 4 marks)*

14. Use either Stoke's theorem or divergence theorem to evaluate the following integral $\iint (curl\,\vec{v}).\vec{n}\,d\sigma$ over any surface whose bounding curve is in the (x, y) plane where

$$\vec{v}=(x-x^2 z)i+(yz^3-y^2)j+(x^2 y-xz)k.$$

15. Prove that a square matrix A can be diagonalized by similarity transformation with a unitary matrix if and only if it is normal.
16. A meteorite falls on the surface of earth from a random direction; it is equally likely to fall on any part of the earth. What is the probability that its place of fall has latitude between 30^0 and 45^0 north?
17. Using Pauli spin matrices $\sigma_x, \sigma_y, \sigma_z$ show that $(\vec{\sigma}.\vec{a})(\vec{\sigma}.\vec{b})=\vec{a}.\vec{b}\,\hat{1}+i\vec{\sigma}.(\vec{a}\times\vec{b})\; where\; \sigma=i\sigma_x+j\sigma_y+k\sigma_z$ and $\vec{a}$and $\vec{b}$ are ordinary vectors.
18. From the generating function derive the recurrence relations

$$H_{n+1}(x)=2n\,H_n(x)-2nH_{n-1}(x)$$
$$H'_n(x)=2nH_{n-1}(x)$$

(3 × 4 = 12)

Part D (Essay)

*(Answer **all questions**. Each question carries 12 marks)*

19. (a) Derive the expression for gradient, divergence and curl in general curvilinear coordinate system. Use the result to find the expression for the same in circular and spherical polar coordinates.

(Or)

(b) Explain the various steps involved in Gram-Schmidt orthogonalization procedure. Show that Legendre polynomials can be generated by this technique by suitably selecting the interval and weight function.

20. (a) Show that the inertia matrix for a single particle of mass m at (x, y, z) has a zero determinant. Explain this result in terms of the invariance of the determinant of a matrix under similarity transformations and a possible rotation of the coordinate system.

(Or)

(b) State and prove the central limit theorem of probability distributions

21. (a) Define Christoffel's symbols of first and second kind. Derive their transformation laws and show that they are not the components of a

tensor; but when linear transformations are assumed, they transform like the components of a tensor.

(Or)

(b) Explain what is meant by a curvature tensor. Show that the curvature tensor may be contracted in two ways; one yielding a zero tensor and the other a symmetric tensor.

22. (a) State and prove the orthogonality properties of Legendre Polynomials

(Or)

(b) Derive Bessel's equation from Legendre equation.

(4 × 12 = 48)